Mixed Review – Quadratic Relations — Answers

Each answer shows the key steps. Readings from a graph will differ a little from mine, so each reading has a range of accepted answers.

1. A girl jumps off a diving board into a pool. The end of the board is 1.5 m above the water. She jumps from the end. Her path is modelled by $y = -\frac{1}{2}(x-1)^2 + 2$. Let x be her horizontal distance from the end of the board, in metres. Let y be her height above the water, in metres.

a) Which feature of the parabola is 1.5: the vertex, a zero, or the y-intercept? Show that the model gives 1.5.

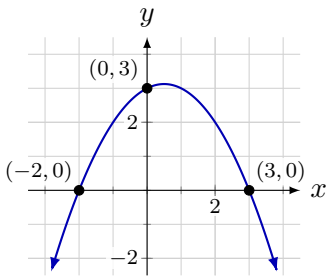
The y-intercept. The end of the board is at $x = 0$: $-\frac{1}{2}(0-1)^2 + 2 = -\frac{1}{2} + 2 = 1.5$.

b) What is her horizontal distance from the end of the board when she reaches the water? Use the step pattern or a table of values.

3 m. At the water, $y = 0$. The vertex is $(1, 2)$ and $a = -\frac{1}{2}$, so the steps go down $\frac{1}{2}$, then $\frac{3}{2}$: $(2, 1.5)$, then $(3, 0)$. So $y = 0$ when $x = 3$.

Review: *Modelling with Vertex Form and Intercepts Form*

2.



Which equation matches this graph? Explain why you chose it. Then explain why each of the other three equations does not match.

A $y = -\frac{1}{2}(x-2)(x+3)$ **B** $y = -\frac{1}{2}(x+2)(x-3)$

C $y = -(x+2)(x-3)$ **D** $y = \frac{1}{2}(x+2)(x-3)$

B. Its zeros are -2 and 3, it opens down ($a < 0$), and at $x = 0$: $-\frac{1}{2}(2)(-3) = 3$.
A has zeros 2 and -3: each zero has the opposite sign to the number in its bracket.
C has the right zeros, but at $x = 0$: $-(2)(-3) = 6$, not 3. The value of a is wrong.
D opens up ($a > 0$), but the graph opens down.

Review: *Intercepts Form*

3. A parabola has zeros -4 and 8. Its maximum value is 9. Find its equation in vertex form. Start by finding the vertex.

The axis of symmetry is midway between the zeros: $x = \frac{-4+8}{2} = 2$. The maximum value is $k = 9$, so the vertex is $(2, 9)$. $y = a(x-2)^2 + 9$. Substitute the point $(8, 0)$: $0 = a(8-2)^2 + 9$, so $36a = -9$ and $a = -\frac{1}{4}$. $y = -\frac{1}{4}(x-2)^2 + 9$

Review: *Intercepts Form* and *Analysing Vertex Form and Equations from Graphs*

4. Ana says: "7^{-1} is less than 7^0. This is because $7^0 = 0$, and 7^{-1} is a negative number." Ana is right that 7^{-1} is less than 7^0. But both of her reasons are wrong. Explain why each reason is wrong. Give the values of 7^{-1} and 7^0.

$7^0 = 1$, not 0. By the exponent law, $7^1 \div 7^1 = 7^0$, and $7 \div 7 = 1$.

7^{-1} is not negative. A negative exponent means "one over": $7^{-1} = \frac{1}{7}$, which is positive.

Review: *Negative and Zero Exponents*

5. A candle burns. Its height was measured every 10 minutes.

Time (min)	Height (mm)	First differences	Second differences
0	200		
		−17	
10	183		2
		−15	
20	168		−2
		−17	
30	151		2
		−15	
40	136		−2
		−17	
50	119		

a) Check the Time column first. Then complete the difference columns. Is the relation linear, quadratic, or neither? The heights were measured. So the differences may not be exactly equal.

Linear. The times go up evenly by 10, so finite differences can be used. The first differences are close to constant: −17 or −15, about −16 each time. The second differences stay close to 0 (2 and −2). So the relation is linear, not quadratic.

b) Predict the height of the candle after 80 minutes. Explain how you found it. Is this an interpolation or an extrapolation?

About 71 mm (accept 68 to 74). The height falls about 16 mm every 10 minutes. From 50 to 80 minutes is 3 more steps of 10: $119 - 3 \times 16 = 71$. It is an **extrapolation**: 80 minutes is beyond the data (0 to 50 minutes).

Review: *Quadratic Relations and Finite Differences* and *Relations and Models*

6. Think about the parabola $y = 2(x - 3)^2 + k$. Do not make a table. Explain each answer.

a) Give one value of k so that the parabola has no zeros.
Any $k > 0$, for example $k = 1$. It opens up and its minimum value, k, is greater than 0. So it never reaches $y = 0$.

b) Give the value of k so that the parabola has exactly one zero.
$k = 0$. The vertex $(3, 0)$ is on the x-axis, so it touches the x-axis once.

c) Give one value of k so that the parabola has two zeros.
Any $k < 0$, for example $k = -2$. The vertex is below the x-axis and the parabola opens up. So it crosses the x-axis twice.

d) Now the parabola is $y = -2(x - 3)^2 + k$. Which values of k give no zeros?
All $k < 0$. It opens down, so k is a maximum value. If the maximum is below the x-axis, it never reaches $y = 0$.

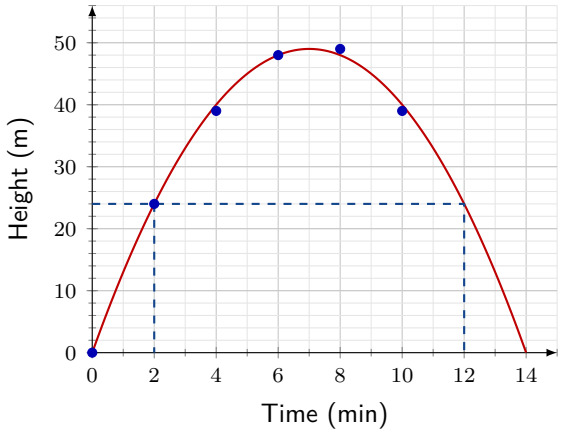
Review: *The Parent Parabola and Vertical Translations*

7. Leo says: "The graph of $y = \frac{1}{3}x^2$ is narrower than the graph of $y = x^2$, because each y-value is divided by 3." Explain Leo's error. Use the point where $x = 3$ in your explanation.

Leo is right that each y-value is divided by 3, but that makes the graph **wider**, not narrower. At $x = 3$, $y = x^2$ gives 9, but $y = \frac{1}{3}x^2$ gives only 3. Each point is closer to the x-axis, so the graph is wider. This is a vertical compression by a factor of $\frac{1}{3}$.

Review: *Transformations of the Quadratic Relation*

8. A child flies a kite. The kite's height was measured every 2 minutes for 10 minutes. A curve of best fit is drawn and continued to 14 minutes.



a) Use the curve to find the two times when the curve gives a height of 24 m.

From 24 on the vertical axis, go across to the curve. Then go down to the time axis: 2 min and 12 min (accept 1.6 to 2.4 and 11.6 to 12.4).

b) The curve gives a height of 0 m at 14 minutes. Is this an interpolation or an extrapolation? Explain why this prediction may be wrong.

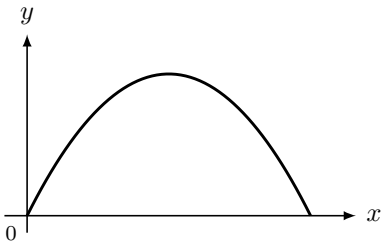
An **extrapolation**: 14 minutes is beyond the data, which stop at 10 minutes. Nothing was measured after 10 minutes, so the pattern may not continue. (For example, the wind can change.)

Review: *Relations and Models*

9. The opening of a large tent has the shape of a parabola. It is modelled by

$$y = -0.4x(x - 5)$$

Let x be the horizontal distance from the left side of the opening, in metres. Let y be the height of the opening, in metres.



a) How high is the opening at its highest point? Show how you know.

2.5 m. The zeros are 0 and 5, so the highest point is midway, at $x = 2.5$: $-0.4(2.5)(2.5 - 5) = 2.5$.

b) A box is 1 m wide. It is pushed through the centre of the opening. What is the greatest height the box can have? Explain.

2.4 m. The centre is $x = 2.5$, so the sides of the box are at $x = 2$ and $x = 3$. Above the box, the opening is lowest at $x = 2$ and $x = 3$. $-0.4(2)(2 - 5) = 2.4$, and by symmetry the height at $x = 3$ is also 2.4 m. So the greatest height is 2.4 m.

Review: *Modelling with Vertex Form and Intercepts Form*

10. The point $(9, 1)$ is on the parabola $y = \frac{1}{4}(x-5)^2 - 3$. Find another point on the parabola with $y = 1$. Use the axis of symmetry. Do not use the equation to calculate. Explain in words why your point must be on the parabola.

$(1, 1)$. The axis of symmetry is $x = 5$. The point $(9, 1)$ is 4 units to the right of the axis. If you fold the graph along the axis, each point lands on its partner. The partner is at the same height, 4 units to the left of the axis: $(1, 1)$.

Review: *Analysing Vertex Form and Equations from Graphs*