Analysing Vertex Form and Equations from Graphs

Adapted from Principles of Mathematics 10, McGraw-Hill Ryerson, 2007.

Read the Recall box, then try the questions. Show your substitution whenever you find a.

Recall: An Equation from a Vertex and One More Point

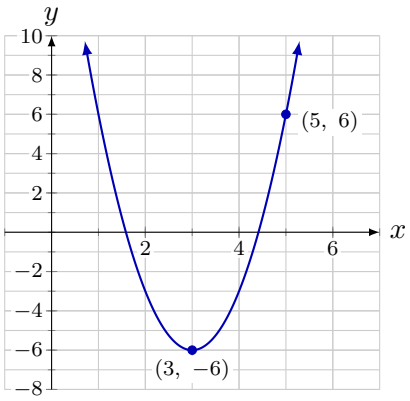
The vertex gives h and k, so write $y = a(x-h)^2 + k$ with only a unknown. Then find a one of two ways.

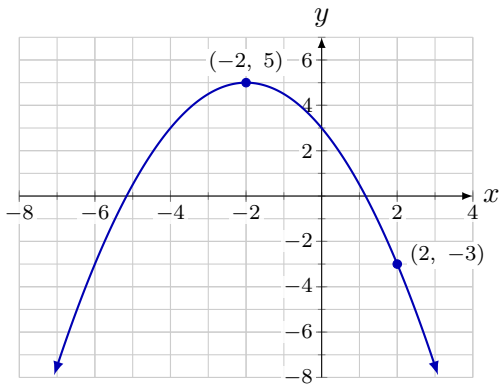
By the step pattern: from the vertex, 1 across goes up (or down) $1a$, the next 1 across $3a$. This needs y-values you can read exactly one step and two steps from the vertex. When they fall between grid lines, substitute a point instead.

By substitution: put the other point into the equation and solve for a. This works for any point.

To find the ***y*-intercept**, substitute $x = 0$. Two points the same distance from the **axis of symmetry**, $x = h$, have the same height: fold the graph along the axis and they land on each other.

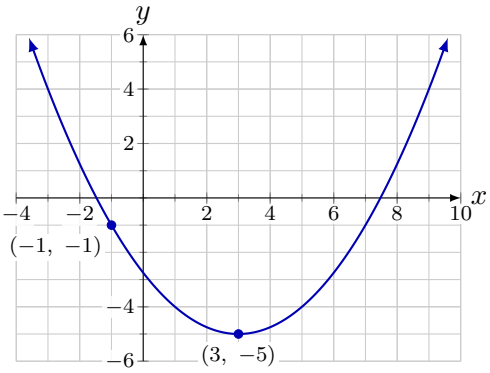
1. Each graph shows a parabola. Write its equation in the form $y = a(x-h)^2 + k$.

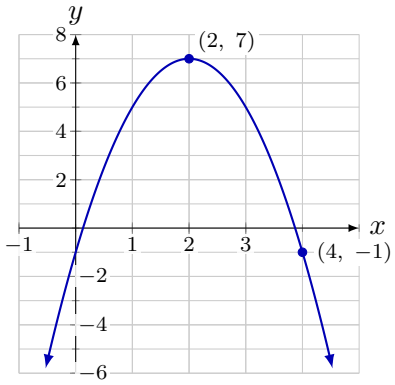




a) $y =$ _____

b) $y =$ _____





c) $y =$ _____

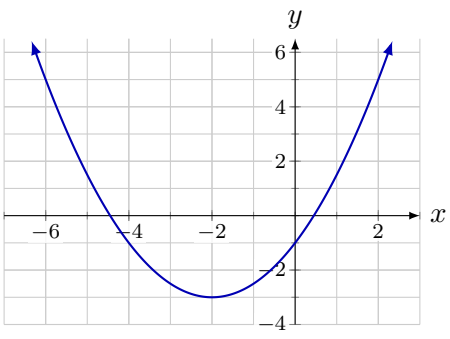
d) $y =$ _____

2. Each part describes a parabola in words. Write its equation in the form $y = a(x - h)^2 + k$.

a) Its vertex is $(-5,\ 2)$. It passes through the point $(-3,\ 10)$.

b) Its vertex is $(-4,\ 3)$. Its y-intercept is -5.

3. The graph shows a parabola.



Which equation fits the graph?

A $y = 2(x + 2)^2 - 3$ **B** $y = \frac{1}{2}(x - 2)^2 - 3$

C $y = \frac{1}{2}(x + 2)^2 - 3$ **D** $y = \frac{1}{2}(x + 2)^2 + 3$

Choose one. Give a reason against each of the others.

4. Here are the equations of three parabolas. Complete the analysis table. For the factor, write the stretch or compression factor compared with $y = x^2$.

Relation	**a)** $y = 3(x+1)^2 - 8$	**b)** $y = -0.5(x-6)^2 + 10$	**c)** $y = 0.2(x-5)^2 + 1$
Vertex			
Axis of symmetry			
Opens			
Stretch or compression factor			
Maximum or minimum value			
y-intercept (show how)			
The other point at the height of the y-intercept			

5. Here is the equation of a parabola:

$$y = -\tfrac{1}{2}(x-4)^2 + 7$$

a) Show by substitution that the point $(1,\ 2.5)$ is on the parabola.

b) Write another point on the parabola with y-value 2.5. Do not substitute.

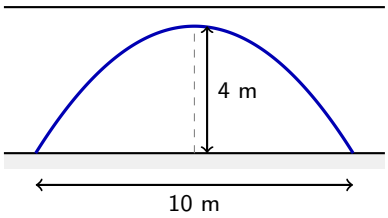
c) The line $x = 4$ is called the axis of symmetry of this parabola. Use your two points to explain why.

6. The table shows values of $y = x^2$ and $y = 3x^2$.

x	x^2	$3x^2$
-2	4	12
-1	1	3
0	0	0
1	1	3
2	4	12

Use the table to explain why $y = 3x^2$ is called a vertical stretch of $y = x^2$ by a factor of 3.

7. A footbridge crosses a stream. The bridge is held up by an arch in the shape of a parabola. The arch is 10 m wide at the ground and 4 m tall at its centre.



a) Put the left end of the arch at $(0,\ 0)$. Write the coordinates of the top of the arch and of its right end.

b) Write an equation for the arch in vertex form.

c) How high is the arch 2 m from its centre?

d) Which values of x make sense for the arch?

8. Work backwards. Write the equation of each parabola in vertex form.

a) It has the same shape as $y = 2x^2$ and opens upward. Its axis of symmetry is $x = -3$. Its y-intercept is 13.

b) It has the same shape as $y = \frac{1}{3}x^2$, but it opens downward. Its axis of symmetry is $x = 6$. Its y-intercept is -2.

Analysing Vertex Form and Equations from Graphs — Hints

Stuck? Read the hint for that question, then go back and try again. Each hint points back to an idea from today's task cards. It does not give you the answer.

Question 1

Recall Cards 1 and 3 (Questions 1 and 3 on the discussion page): the two ways we found a from a graph. Card 3 shows what to do when the steps from the vertex cannot be read exactly.

Question 2

Recall Card 4 (Question 4 on the discussion page), where nobody had drawn the parabola. Sketch what the words say on scrap paper first.

Question 3

Recall Cards 1 and 3 (Questions 1 and 3 on the discussion page): how the vertex and the steps show on a graph. Each wrong equation fails on a different feature of the graph.

Question 4

Recall from the Transformations of the Quadratic Relation summary how a, h, and k show in the equation. For the last two rows, recall Card 7 (Question 7 on the discussion page).

Question 5

Recall Cards 5 and 6 (Questions 5 and 6 on the discussion page), where we named a point with no arithmetic and said why the axis of symmetry has its name.

Question 6

Recall Card 2 (Question 2 on the discussion page), where two students described $y = 4x^2$ in two ways.

Question 7

Recall Card 4 (Question 4 on the discussion page): the vertex and one more point give the equation. Which points on the arch do you know exactly?

Question 8

Recall from the Transformations of the Quadratic Relation summary what each of a, h, and k does, and recall Card 7 (Question 7 on the discussion page) for the y-intercept. Here you know some features and work back to the equation.