Applications of Linear Systems — Full Solutions

The complete working for each question: the set up, the solving, a left-side/right-side check, and the answer in a sentence. Values that have been substituted are in red; balancing steps are in light red; notes are in blue.

Question 1

At a sale in a retro music store, all CDs are one price and all tapes are another price. Three CDs and two tapes cost \$72. One CD and three tapes cost \$52. What are the prices of one CD and one tape?

Let x be the cost of one CD.

Let y be the cost of one tape.

$$\begin{aligned} 3x + 2y &= 72 && \text{(A)} \\ x + 3y &= 52 && \text{(B)} \\ \\ 3x + 2y &= 72 && \text{(A)} \\ 3x + 9y &= 156 && \text{(B)} \times 3 \\ \hline -7y &= -84 && \ominus \\ y &= \frac{-84}{-7} \\ y &= 12 \end{aligned}$$

Sub $y = 12$ into (B)

$$\begin{aligned} x + 3y &= 52 \\ x + 3(12) &= 52 \\ x + 36 &= 52 \\ x &= 52 - 36 \\ x &= 16 \end{aligned}$$

Check $\left(\overset{x}{16},\ \overset{y}{12}\right)$

... in (A)

$$\begin{aligned} \text{LS} &= 3x + 2y \\ &= 3(16) + 2(12) \\ &= 48 + 24 \\ &= 72 \\ &= \text{RS} \end{aligned}$$

LS = RS

... in (B)

$$\begin{aligned} \text{LS} &= x + 3y \\ &= 16 + 3(12) \\ &= 16 + 36 \\ &= 52 \\ &= \text{RS} \end{aligned}$$

LS = RS

$\therefore$ the cost of one CD is \$16 and the cost of one tape is \$12.

Question 2

A sports club charges an initiation fee and a monthly fee. At the end of 5 months, a member had paid a total of \$170. At the end of 10 months, she had paid a total of \$295. What was the initiation fee?

Let x be the initiation fee in dollars.

Let y be the monthly fee in dollars.

$$\begin{aligned} x + 5y &= 170 \quad \text{(A)} \\ x + 10y &= 295 \quad \text{(B)} \\ \hline -5y &= -125 \quad (-) \\ y &= \frac{-125}{-5} \\ y &= 25 \end{aligned}$$

Sub $y = 25$ into (A)

$$\begin{aligned} x + 5y &= 170 \\ x + 5(25) &= 170 \\ x &= 170 - 125 \\ x &= 45 \end{aligned}$$

Check $\begin{pmatrix} x & y \\ 45, & 25 \end{pmatrix}$

... in (A)

$$\begin{aligned} \text{LS} &= x + 5y \qquad \text{RS} = 170 \\ &= 45 + 5(25) \\ &= 45 + 125 \\ &= 170 \\ &\text{LS} = \text{RS} \end{aligned}$$

... in (B)

$$\begin{aligned} \text{LS} &= x + 10y \qquad \text{RS} = 295 \\ &= 45 + 10(25) \\ &= 45 + 250 \\ &= 295 \\ &\text{LS} = \text{RS} \end{aligned}$$

$\therefore$ the initiation fee is \$45.

Question 3

A tennis club charges an annual fee and an hourly fee for court time. One year, one member played for 39 hours and paid \$384. Another member played for 51 hours and paid \$456. Determine the annual fee and the hourly fee.

Let x be the annual fee.

Let y be the hourly fee.

$$\begin{aligned} x + 39y &= 384 \quad \text{(A)} \\ x + 51y &= 456 \quad \text{(B)} \\ \hline -12y &= -72 \quad \text{(−)} \\ y &= \frac{-72}{-12} \\ y &= 6 \end{aligned}$$

Sub $y = 6$ into (A)

$$\begin{aligned} x + 39y &= 384 \\ x + 39(6) &= 384 \\ x + 234 &= 384 \\ x &= 384 - 234 \\ x &= 150 \end{aligned}$$

Check $\left(\overset{x}{150},\ \overset{y}{6}\right)$

... in (A)

$$\begin{aligned} \text{LS} &= x + 39y \qquad \text{RS} = 384 \\ &= (150) + 39(6) \\ &= 150 + 234 \\ &= 384 \\ &\text{LS} = \text{RS} \end{aligned}$$

... in (B)

$$\begin{aligned} \text{LS} &= x + 51y \qquad \text{RS} = 456 \\ &= 150 + 51(6) \\ &= 150 + 306 \\ &= 456 \\ &\text{LS} = \text{RS} \end{aligned}$$

$\therefore$ the annual fee is \$150 and the hourly fee is \$6.

Question 4

Three footballs and one soccer ball cost \$155. Two footballs and three soccer balls cost \$220. Determine the cost of one football and the cost of one soccer ball.

Let x be the cost of one football.

Let y be the cost of one soccer ball.

$$\begin{aligned} 3x + y &= 155 && \text{(A)} \\ 2x + 3y &= 220 && \text{(B)} \\ 9x + 3y &= 465 && \text{(A)} \times 3 \\ 2x + 3y &= 220 && \text{(B)} \\ \hline 7x &= 245 && \ominus \\ x &= \frac{245}{7} \\ x &= 35 \end{aligned}$$

Sub $x = 35$ into (A)

$$\begin{aligned} 3x + y &= 155 \\ 3(35) + y &= 155 \\ 105 + y &= 155 \\ y &= 155 - 105 \\ y &= 50 \end{aligned}$$

Check $\left(\overset{x}{35},\ \overset{y}{50}\right)$

... in (A)

$$\begin{aligned} \text{LS} &= 3x + y \\ &= 3(35) + 50 \\ &= 155 \end{aligned} \qquad \text{RS} = 155$$

$$\text{LS} = \text{RS}$$

... in (B)

$$\begin{aligned} \text{LS} &= 2x + 3y \\ &= 2(35) + 3(50) \\ &= 70 + 150 \\ &= 220 \end{aligned} \qquad \text{RS} = 220$$

$$\text{LS} = \text{RS}$$

$\therefore$ the cost of one football is \$35 and the cost of one soccer ball is \$50.

Question 5

For the school play, one adult ticket cost \$5.00 and one student ticket cost \$3.00. Twice as many student tickets as adult tickets were sold. The total receipts were \$1650. How many of each kind of ticket were sold?

Let x be the number of student tickets sold.

Let y be the number of adult tickets sold.

- "y" adult tickets sold.
- student tickets sold twice as many as adult tickets
- So...

$$x = 2y \quad \text{(A)}$$
$$3x + 5y = 1650 \quad \text{(B)}$$

$x = 2y$ (student tickets sold; adult tickets sold)

Sub $x = 2y$ into (B)

$$\begin{aligned} 3x + 5y &= 1650 \\ 3(2y) + 5y &= 1650 \\ 6y + 5y &= 1650 \\ 11y &= 1650 \\ y &= \frac{1650}{11} \\ y &= 150 \end{aligned}$$

Sub $y = 150$ into (A)

$$\begin{aligned} x &= 2y \\ x &= 2(150) \\ x &= 300 \end{aligned}$$

Check $\left(\overset{x}{300},\ \overset{y}{150}\right)$

... in (A)

$$\begin{aligned} \text{LS} &= x & \text{RS} &= 2y \\ &= 300 & &= 2(150) \\ & & &= 300 \\ & \text{LS} = \text{RS} & & \end{aligned}$$

... in (B)

$$\begin{aligned} \text{LS} &= 3x + 5y & \text{RS} &= 1650 \\ &= 3(300) + 5(150) & & \\ &= 900 + 750 & & \\ &= 1650 & & \\ & \text{LS} = \text{RS} & & \end{aligned}$$

$\therefore$ 300 student tickets were sold and 150 adult tickets were sold.

Question 6

An American football stadium has 20 000 seats between the goal lines and 5000 in the end zones. An end-zone seat is \$5 cheaper than one between the goal lines. For one game, the revenue was \$350 000 when all seats were sold. What is the cost of each type of seat?

Let x be the cost of a seat between the goal lines.

Let y be the cost of a seat in the end zones.

$$y = x - 5 \quad \text{(A)}$$
$$20\,000x + 5000y = 350\,000 \quad \text{(B)}$$

Sub $y = x - 5$ into (B)

$$\begin{aligned} 20\,000x + 5000y &= 350\,000 \\ 20\,000x + 5000(x - 5) &= 350\,000 \\ 20\,000x + 5000x - 25\,000 &= 350\,000 \\ 25\,000x &= 350\,000 + 25\,000 \\ 25\,000x &= 375\,000 \\ x &= \frac{375\,000}{25\,000} \\ x &= 15 \end{aligned}$$

Sub $x = 15$ into (A)

$$\begin{aligned} y &= x - 5 \\ y &= 15 - 5 \\ y &= 10 \end{aligned}$$

Check $\left(\overset{x}{15},\ \overset{y}{10}\right)$

... in (A)

$$\begin{aligned} \text{LS} &= y \\ &= 10 \end{aligned} \qquad \begin{aligned} \text{RS} &= x - 5 \\ &= 15 - 5 \\ &= 10 \end{aligned}$$

$\text{LS} = \text{RS}$

... in (B)

$$\begin{aligned} \text{LS} &= 20\,000x + 5000y \\ &= 20\,000(15) + 5000(10) \\ &= 350\,000 \end{aligned} \qquad \text{RS} = 350\,000$$

$\text{LS} = \text{RS}$

$\therefore$ a seat between the goal lines costs \$15 and a seat in the end zones costs \$10.

Question 7

When 20 bolts are placed in a box, the total mass is 340 g. When there are 48 bolts in the box, the total mass is 760 g. Determine the mass of the box and the mass of each bolt.

Let x be the mass of the box.

Let y be the mass of a bolt.

$$\begin{aligned} x + 20y &= 340 \quad \text{(A)} \\ x + 48y &= 760 \quad \text{(B)} \\ \hline -28y &= -420 \quad \ominus \\ y &= \frac{-420}{-28} \\ y &= 15 \end{aligned}$$

Sub $y = 15$ in (A)

$$\begin{aligned} x + 20y &= 340 \\ x + 20(15) &= 340 \\ x + 300 &= 340 \\ x &= 340 - 300 \\ x &= 40 \end{aligned}$$

Check $\begin{pmatrix} x & y \\ 40, & 15 \end{pmatrix}$

... in (A)

$$\begin{aligned} \text{LS} &= x + 20y \qquad \text{RS} = 340 \\ &= 40 + 20(15) \\ &= 40 + 300 \\ &= 340 \\ &\text{LS} = \text{RS} \end{aligned}$$

... in (B)

$$\begin{aligned} \text{LS} &= x + 48y \qquad \text{RS} = 760 \\ &= 40 + 48(15) \\ &= 40 + 720 \\ &= 760 \\ &\text{LS} = \text{RS} \end{aligned}$$

$\therefore$ the box has a mass of 40 g and each bolt has a mass of 15 g.

Question 8

A crate of 36 grapefruit has a total mass of 4 kg. When 12 grapefruit are removed, the total mass is 3 kg. Determine the mass of the crate and the mass of one grapefruit.

Let x be the mass of the crate.

Let y be the mass of one grapefruit.

$$\begin{aligned} x + 36y &= 4 \quad \text{(A)} \\ x + 24y &= 3 \quad \text{(B)} \\ \hline 12y &= 1 \quad (-) \\ y &= \frac{1}{12} \end{aligned}$$

Sub $y = \frac{1}{12}$ into (B)

$$\begin{aligned} x + 24y &= 3 \\ x + 24\left(\tfrac{1}{12}\right) &= 3 \\ x + 2 &= 3 \\ x &= 3 - 2 \\ x &= 1 \end{aligned}$$

Check $\begin{pmatrix} x & y \\ 1, & \frac{1}{12} \end{pmatrix}$

... in (A)

$$\begin{aligned} \text{LS} &= x + 36y \qquad \text{RS} = 4 \\ &= 1 + 36\left(\tfrac{1}{12}\right) \\ &= 1 + 3 \\ &= 4 \\ &\text{LS} = \text{RS} \end{aligned}$$

... in (B)

$$\begin{aligned} \text{LS} &= x + 24y \qquad \text{RS} = 3 \\ &= 1 + 24\left(\tfrac{1}{12}\right) \\ &= 1 + 2 \\ &= 3 \\ &\text{LS} = \text{RS} \end{aligned}$$

$\therefore$ the mass of the crate is 1 kg and the mass of one grapefruit is $\frac{1}{12}$ kg or about 0.083 kg.