Get Ready for Quadratic Relations

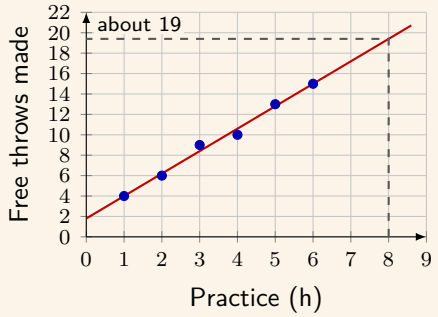
Adapted from Principles of Mathematics 10, McGraw-Hill Ryerson, 2007.

Read each Recall box, then try the questions after it. Plot on the grids printed here.

Recall: Scatter Plots

A coach set six players different hours of practice, then each took 20 free throws. The results, as (hours, free throws made): (1, 4), (2, 6), (3, 9), (4, 10), (5, 13), (6, 15).

The coach *controlled* the hours, so practice time is the **independent variable** (horizontal axis); free throws made is the **dependent variable**. The points rise and lie close to a line: the relation is **positive**, **strong**, and **linear**. If the trend continues, 8 hours of practice gives about 19 free throws. Eight hours is beyond the data, so this is an **extrapolation**.



1. I measured a sunflower seedling's height every day for 14 days after it broke through the soil.

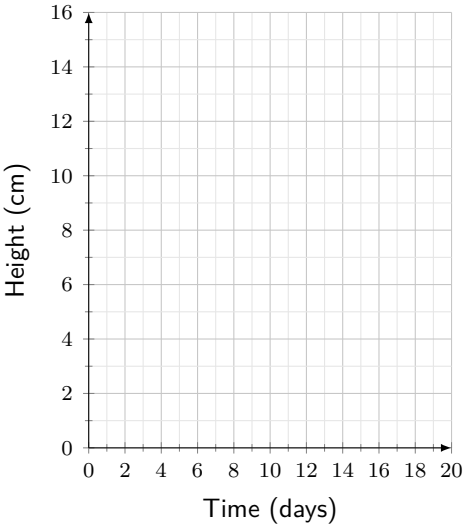
Time (days)	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Height (cm)	0.4	0.7	1.1	1.4	2.0	2.5	3.0	3.8	4.5	5.1	6.1	7.0	7.8	9.0

a) Name the independent and dependent variables.

_____

_____

b) Plot the data on the grid.



c) Describe how height changes as time changes.

_____

_____

d) Draw a curve of best fit. Use it to predict the height after 18 days. Interpolation or extrapolation? How do you know?

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2. I measured the hand span (thumb tip to little-finger tip, hand stretched wide) and the height of 12 students.

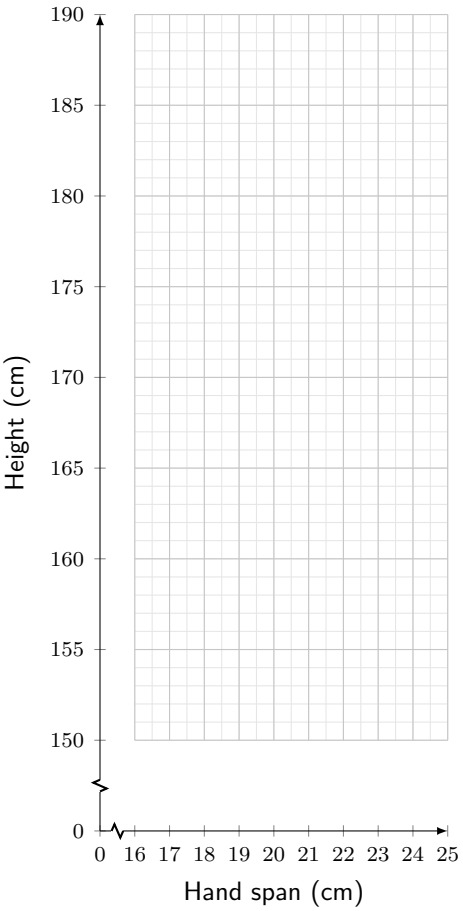
Hand span (cm)	Height (cm)
20.5	170
18.5	158
22.0	176
19.8	171
23.5	177
19.0	165
21.5	167
24.0	184
20.0	164
22.5	172
19.5	160
23.0	180

a) Suppose you want to estimate a student's height from their hand span. Nobody controlled either quantity. Which is the independent variable, and which is the dependent variable? Explain your choice.

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b) Plot the data on the grid. The zigzag on each axis means the scale jumps: from 0 straight to 16 across, and from 0 straight to 150 up. I left out the empty part so the points spread across the grid.



c) Describe how height changes as hand span changes.

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_____

d) Draw a line of best fit. Use it to predict the height of a student with a hand span of 21 cm. Is this an interpolation or an extrapolation? How do you know?

_____

_____

3. A relation can be described in words, as a table of values, as an equation, and as a graph. Here is one in words: *at a party, every guest shakes hands once with every other guest.* The relation links the number of guests to the number of handshakes.

Guests	Hand-shakes
1	
2	
3	
4	
5	
6	
7	

a) As a table of values: complete the table. Try drawing dots for guests and lines for handshakes.

b) As a graph: plot the table on the grid. Should you join the points with a curve? Explain.

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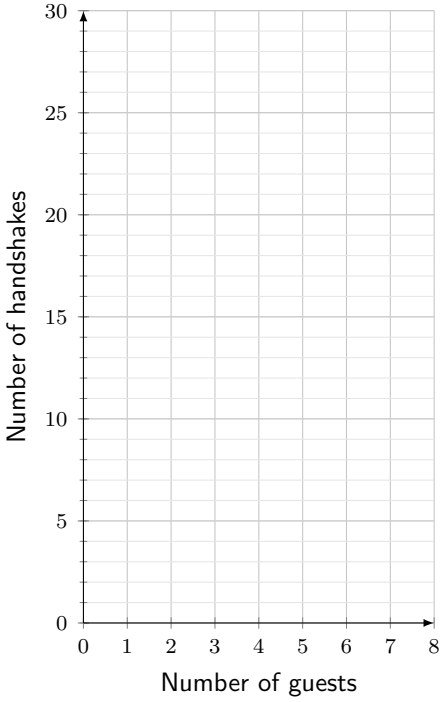
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c) In words again: describe how the number of handshakes changes as guests arrive. How does the table show the shape? How does the graph?

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d) Predict the number of handshakes for 10 guests. Is this an interpolation or an extrapolation? Why can you trust this prediction more than the one in Question 1 d)?

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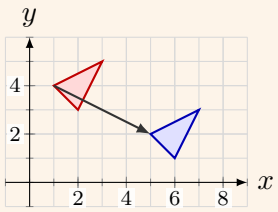
e) The fourth way, an equation, is still missing. What would an equation let you do that the table cannot?

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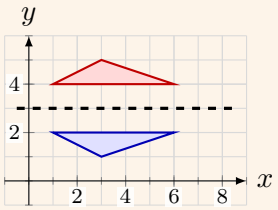
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Recall: Translations and Reflections

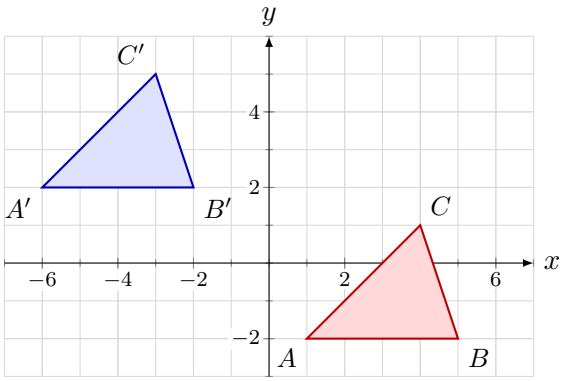
A **translation** slides a figure: every point moves the same distance in the same direction. Here, 4 right and 2 down: $(x,\ y) \to (x+4,\ y-2)$.



A **reflection** flips a figure in a **line of reflection**: each point and its image are the same distance from the line, on opposite sides. Here, the line is $y = 3$.



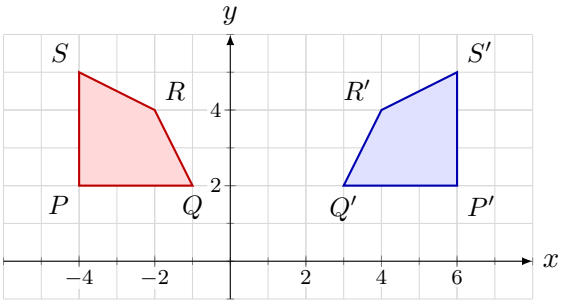
4. Triangle ABC (red) was translated to triangle $A'B'C'$ (blue).



a) Write the coordinates of each vertex and its image.

b) Describe the translation in words and as a rule $(x,\ y) \to (\ \ ,\ \ )$.

5. Quadrilateral $PQRS$ (red) was reflected to quadrilateral $P'Q'R'S'$ (blue).



a) Draw the line of reflection on the grid.

b) Explain how you know your line is in the right place. Use one vertex and its image.

Recall: Powers and the Exponent Laws

A **power** is a number multiplied by itself: $2^5 = 2 \times 2 \times 2 \times 2 \times 2$ has **base** 2 and **exponent** 5, the number of factors. Powers with the same base simplify by three laws.

Multiply: add the exponents	**Divide: subtract the exponents**	**Power of a power: multiply**
$x^a \times x^b = x^{a+b}$	$x^a \div x^b = x^{a-b}$	$(x^a)^b = x^{a \times b}$
$6^2 \times 6^5 = 6^{2+5} = 6^7$	$7^6 \div 7^2 = 7^{6-2} = 7^4$	$(4^3)^2 = 4^{3 \times 2} = 4^6$

6. Write each as one power.

a) $2^4 \times 2^7$

b) $5^7 \div 5^2$

c) $(-3)^4 \times (-3)^5$

d) $\left(\frac{1}{2}\right)^4 \times \left(\frac{1}{2}\right)^6$

e) $(2^3)^4$

f) $(-3)^{11} \div (-3)^3$

7. Each part needs two of the laws. Write each as one power, then evaluate it.

a) $2^5 \times 2^4 \div 2^6$

b) $(5^3)^2 \div 5^4$

c) $(-3)^4 \times (-3)^5 \div (-3)^6$

d) $\left(\frac{1}{2}\right)^3 \times \left(\left(\frac{1}{2}\right)^2\right)^4$

e) $5^9 \div (5^2 \times 5^3)$

f) $((-3)^2)^4 \div (-3)^3$

Get Ready for Quadratic Relations — Final Answers

Adapted from Principles of Mathematics 10, McGraw-Hill Ryerson, 2007.

Final answers only. Lines of best fit drawn by eye will differ a little from mine, so each prediction has a range of accepted answers.

Scatter plots

1. I measured a sunflower seedling's height every day for 14 days after it broke through the soil.

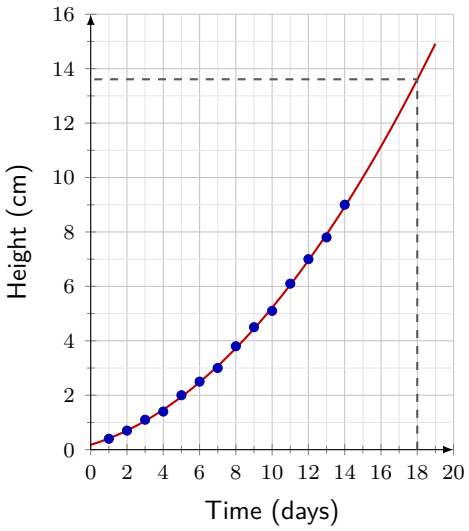
Time (days)	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Height (cm)	0.4	0.7	1.1	1.4	2.0	2.5	3.0	3.8	4.5	5.1	6.1	7.0	7.8	9.0

a) Independent: time (days). Dependent: height (cm). I chose when to measure; the height depends on the time.

b) See the grid.

c) Height increases, and it increases faster as time goes on. Positive, strong (the points follow a smooth pattern closely), and non-linear: the points bend upward.

d) About 13.5 cm (accept 12.5 cm to 15 cm). Extrapolation: 18 days is beyond the data, which stop at 14 days.



2. I measured the hand span (thumb tip to little-finger tip, hand stretched wide) and the height of 12 students.

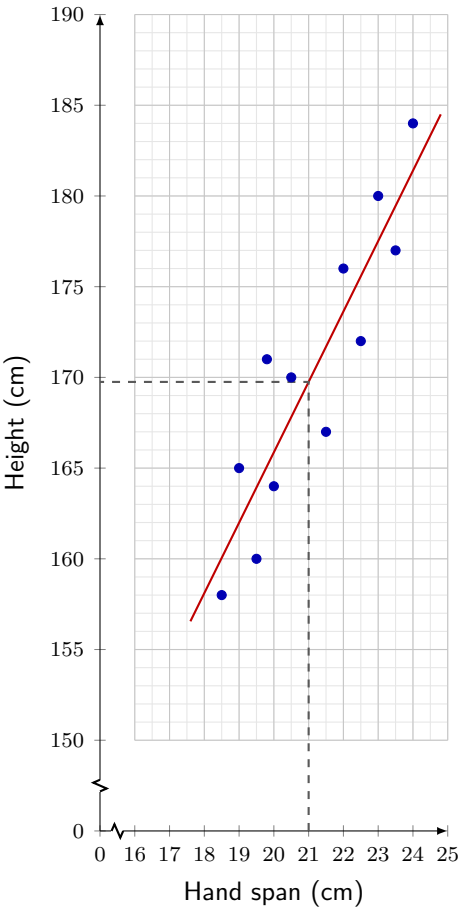
Hand span (cm)	Height (cm)
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19.8	171
23.5	177
19.0	165
21.5	167
24.0	184
20.0	164
22.5	172
19.5	160
23.0	180

a) Independent: hand span (cm). Dependent: height (cm). Neither was controlled, so the prediction we want decides: we predict height *from* hand span.

b) See the grid.

c) As hand span increases, height tends to increase. Positive, moderate to strong (scattered about a line, but following it), and linear.

d) About 170 cm (accept 166 cm to 174 cm). Interpolation: 21 cm lies inside the data, which run from 18.5 cm to 24 cm.



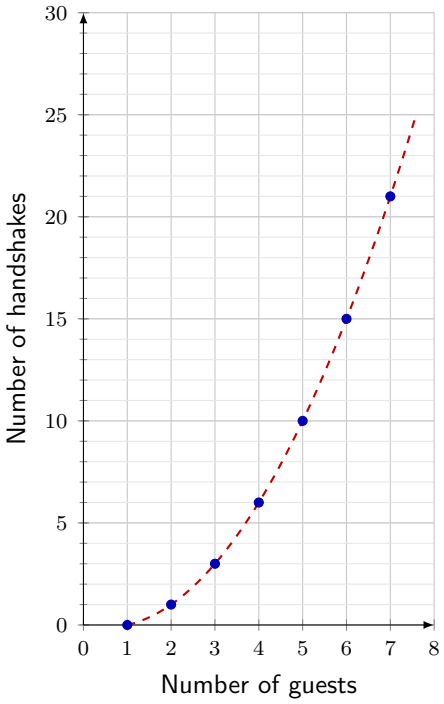
3. A relation can be described in words, as a table of values, as an equation, and as a graph. Here is one in words: *at a party, every guest shakes hands once with every other guest.* The relation links the number of guests to the number of handshakes.

Guests	Hand-shakes
1	0
2	1
3	3
4	6
5	10
6	15
7	21

a) See the table: 0, 1, 3, 6, 10, 15, 21. Each new guest shakes hands with every guest already there, so the jumps are 1, 2, 3, 4, 5, 6.

b) See the grid. Guests come in whole numbers, so only the points have meaning; a dashed curve may show the trend. Accept either answer with that reason.

c) As guests increase, handshakes increase, by more each time: positive and non-linear. The table's jumps grow (1, 2, 3, . . .) instead of staying the same; the graph bends upward. The points lie exactly on the pattern, because a rule makes them, not measurements.

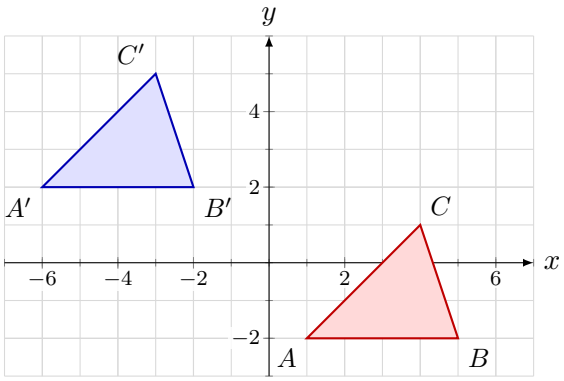


d) 45 handshakes (21 + 7 + 8 + 9). Extrapolation: 10 guests is beyond the table. I can trust it because the rule in words says exactly how the pattern continues; the seedling might stop growing.

e) Find the handshakes for any number of guests, say 50, in one step, without building the table up one row at a time. Equations for curves like this one are what this thread builds.

Translations and reflections

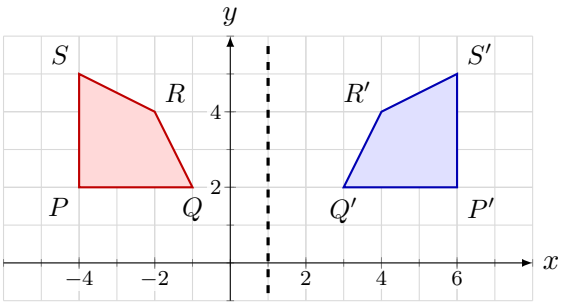
4. Triangle ABC (red) was translated to triangle $A'B'C'$ (blue).



a) $A(1,-2) \to A'(-6,2)$
$B(5,-2) \to B'(-2,2)$
$C(4,1) \to C'(-3,5)$

b) 7 units left and 4 units up;
$(x,\ y) \to (x-7,\ y+4)$

5. Quadrilateral $PQRS$ (red) was reflected to quadrilateral $P'Q'R'S'$ (blue).



a) The vertical line $x = 1$ (dashed).

b) Each vertex and its image are the same distance from the line, on opposite sides: $P(-4,2)$ is 5 units left of $x = 1$ and $P'(6,2)$ is 5 units right of it.

Powers

6. a) 2^{11} b) 5^5 c) $(-3)^9$

d) $\left(\frac{1}{2}\right)^{10}$ e) 2^{12} f) $(-3)^8$

7. a) $2^3 = 8$ b) $5^2 = 25$ c) $(-3)^3 = -27$

d) $\left(\frac{1}{2}\right)^{11} = \frac{1}{2048}$ e) $5^4 = 625$ f) $(-3)^5 = -243$