Horizontal Translations and Vertex Form

Adapted from Principles of Mathematics 10, McGraw-Hill Ryerson, 2007.

Read the Recall box, then try the questions. Sketch on the grids printed here.

Recall: Vertex Form, $y = a(x - h)^2 + k$

Each number moves or reshapes the graph of $y = x^2$:

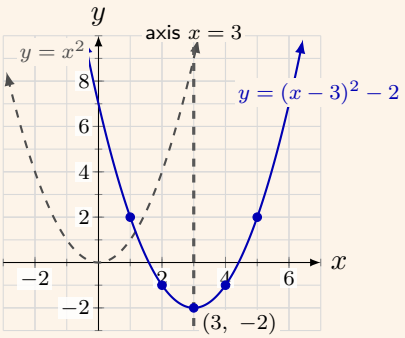
a: a vertical stretch ($a > 1$ or $a < -1$) or compression (a between -1 and 1, not 0); a reflection in the x-axis when $a < 0$, so the parabola opens down.

h: a **horizontal translation**, h units right when $h > 0$, left when $h < 0$. Watch the sign: $y = (x+4)^2$ is $y = (x-(-4))^2$, so $h = -4$, 4 units *left*.

k: a vertical translation, k units up when $k > 0$, down when $k < 0$.

The **vertex** is $(h,\ k)$ and the **axis of symmetry** is $x = h$. The vertex value k is a minimum when $a > 0$ and a maximum when $a < 0$.

To sketch: plot the vertex, then step from it 1 across and $1a$ up, 1 more across and $3a$ more, then $5a$, on each side. When $a < 0$ the steps go down.



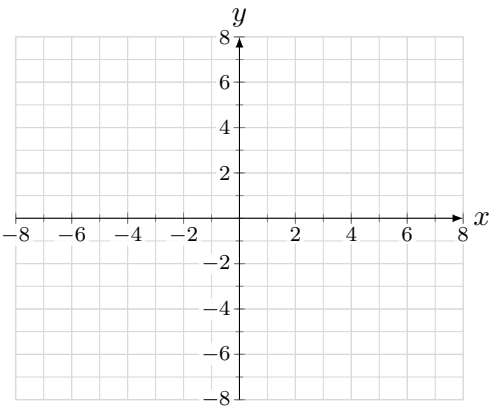
3 right and 2 down; steps 1, 3 up from $(3,\ -2)$.

1. Complete the table for each relation. For the factor, write the stretch or compression factor compared with $y = x^2$, or "none" when $a = 1$ or $a = -1$.

Property	a) $y = (x+5)^2 - 1$	b) $y = -3(x-4)^2 + 7$	c) $y = \frac{1}{4}(x-6)^2$	d) $y = -(x+1.5)^2 - 8$
Vertex				
Axis of symmetry				
Direction of opening				
Stretch or compression factor				
Maximum or minimum value				

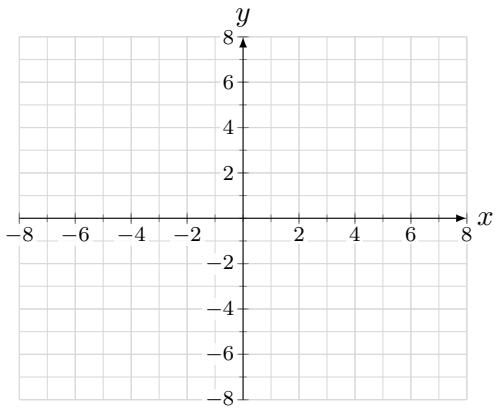
2. Sketch each relation by the step pattern, starting from its vertex. State the vertex.

a) $y = (x-3)^2 - 4$



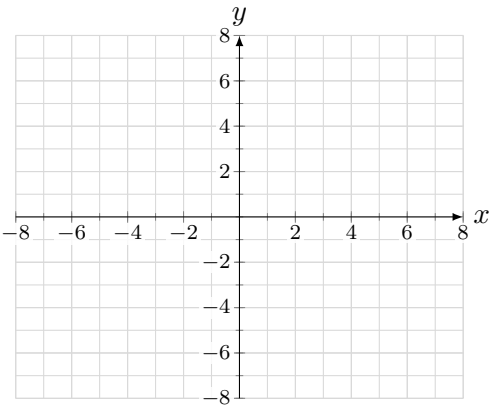
Vertex _____

b) $y = -(x+2)^2 + 6$



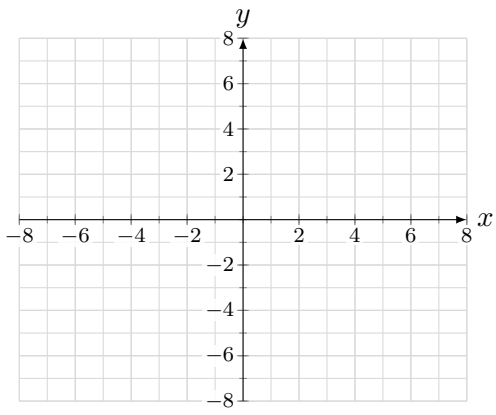
Vertex _____

c) $y = 2(x-1)^2 - 7$



Vertex _____

d) $y = -\frac{1}{2}(x+3)^2 + 2$



Vertex _____

3. Write an equation for each parabola.

a) The graph of $y = x^2$ translated 5 units right. _____

b) The graph of $y = x^2$ translated 6 units left. _____

c) The graph of $y = x^2$ translated 7 units left and 2 units down. _____

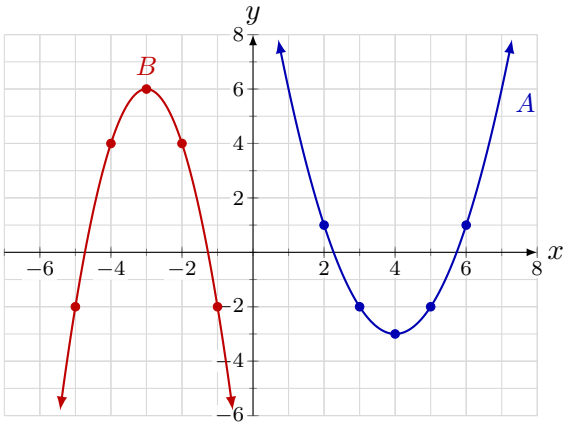
d) The graph of $y = x^2$ translated 3 units right and 10 units up. _____

e) The graph of $y = x^2$ reflected in the x-axis, then translated 6 units right and 1 unit down. _____

f) The graph of $y = x^2$ stretched vertically by a factor of 3, with its vertex moved to $(-2,\ 5)$. _____

g) A parabola with vertex $(8,\ -4)$ that opens downward and is compressed vertically by a factor of 0.5. _____

4. Each parabola is given only as a graph. The marked points sit on grid corners.



a) Parabola A has the same shape as $y = x^2$. Write its equation.

_____

b) Use the step pattern to find a for parabola B. Then write its equation.

_____

_____

c) Check each equation with one marked point that is not the vertex.

_____

_____

5. This relation is given only as a table of values.

x	y
-6	10
-5	5
-4	2
-3	1
-2	2
-1	5
0	10

a) Which row is the vertex? How does the table show it? State the axis of symmetry.

b) Use the steps from the vertex to find a. Write the equation in vertex form, and check it with one row.

c) Describe the graph in words, compared with $y = x^2$.

6. Jordan says the graph of $y = (x + 8)^2 + 3$ is the graph of $y = x^2$ translated 8 units right and 3 units up. Do you agree? Explain, using the vertex.

7. A relation can be given in words, as a table of values, as an equation, and as a graph. This parabola is given as an equation. Give it the other three ways.

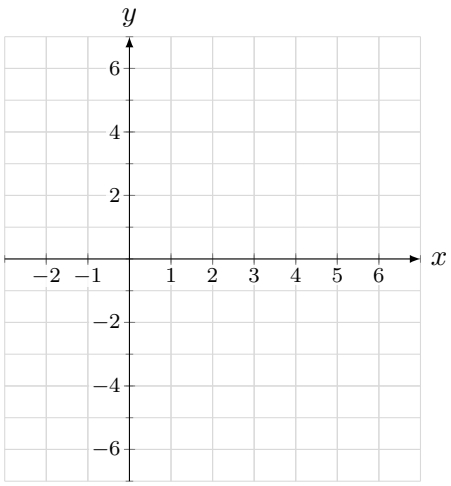
$$y = -(x-2)^2 + 4$$

a) In words: how is it made from $y = x^2$? Which way does it open, where is its vertex, and what is its maximum or minimum value?

b) As a table of values: complete the table.

c) As a graph: plot the points and sketch the parabola on the grid.

x	y
-1	
0	
1	
2	
3	
4	
5	



Try it with technology

I have put a GeoGebra sketch of $y = a(x-h)^2 + k$ online, with a slider for each of a, h, and k:

`russellgordon.ca/lcs/2026-27/mpm2de/sketches/quadratic-transformations`

Work on it on your own. First drag one slider at a time and watch what moves. Then set the sliders to match each relation in Question 2 and compare the graph with your sketch. Last, set them to match your equations in Question 3 and check that each graph does what the words say.

Horizontal Translations and Vertex Form — Hints

Stuck? Read the hint for that question, then go back and try again. Each hint points back to an idea from today's discussion. It does not give you the answer.

Question 1

Recall Card 6 (Question 6 on the discussion page). The vertex is (h, k), and the axis of symmetry is $x = h$. Watch the sign of h when the bracket has a plus sign. The sign of a tells you the direction of opening, and whether k is a maximum or a minimum. The size of a gives the stretch or compression factor.

Question 2

Recall the step pattern from Card 5 (Question 5 on the discussion page) and the summary's steps for sketching. Start at the vertex. Recall from Day 4 how the steps change with a.

Question 3

Recall Card 3 (Question 3 on the discussion page). $y = (x - h)^2$ moves the graph right when $h > 0$ and left when $h < 0$. Watch the sign of h in the bracket. For e) to g), recall from Card 6 what the sign and the size of a each do.

Question 4

For a), read the vertex, and recall from Card 6 (Question 6 on the discussion page) that the vertex is (h, k). Recall what the same shape as $y = x^2$ tells you about a. For b), use the step pattern $1a$, $3a$, $5a$ from the vertex. For c), substitute a point, as we did in Card 5.

Question 5

Recall what the axis of symmetry does to a parabola. Look for that in the table. For b), use the summary's $y = 2(x - 1)^2 - 3$ four ways as a model, and read the steps $1a$, $3a$ from the vertex row. For c), recall what a, h, and k each do.

Question 6

Recall the summary's note "Watch the sign of h". Find the vertex first. Then compare it with what Jordan says.

Question 7

Recall the summary's $y = 2(x-1)^2-3$ four ways. Use it as a model for each part. Check your graph with the step pattern from the vertex.