Modelling with Both Forms

Adapted from Principles of Mathematics 10, McGraw-Hill Ryerson, 2007.

Read the Recall box, then try the questions. Show your substitution whenever you find a.

Recall: Choosing the Form

Vertex form, $y = a(x - h)^2 + k$, suits a situation where you know the highest or lowest point: h tells you where (or when) it happens and k how high it is.
Intercepts form, $y = a(x - r)(x - s)$, suits a situation where you know where the path or the arch meets the ground: the zeros r and s.

Either way, one more point gives a. The sign of a tells you whether the parabola opens up or down. Two points the same distance from the axis of symmetry have the same height.

A model only describes its situation inside the values it was made from. A prediction inside those values (**interpolation**) can be trusted more than one outside them (**extrapolation**).

1. A ball is thrown up from a balcony 3 m above the ground. It reaches its greatest height, 23 m, 2 s after it is thrown. Let x be the time since the ball was thrown, in seconds, and y its height, in metres.

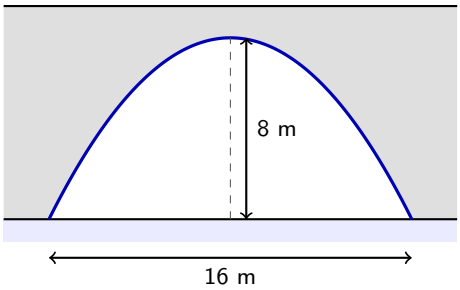
a) Write an equation for the height of the ball.

b) What do h and k tell you about the ball? What does the sign of a tell you?

c) Find the height of the ball 0.5 s after it is thrown.

d) At what other time is the ball at the same height as in c)? Find the time without substituting.

2. A stone bridge crosses a river. The underside of the bridge is a parabolic arch. At the water, the arch is 16 m wide. Its highest point is 8 m above the water.



a) Put the left end of the arch at (0, 0), at the water. Write the coordinates of the right end and of the top of the arch.

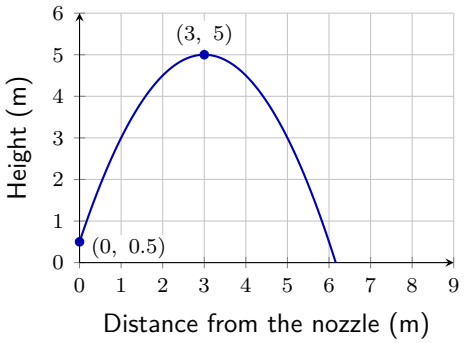
b) Write an equation for the arch in intercepts form.

c) What do the two zeros tell you about the arch?

d) How high is the arch 4 m from its left end?

e) At what other distance from the left end is the arch this high? Do not substitute.

3. A fountain in a park shoots a jet of water. The path of the water is a parabola. The water leaves the nozzle 0.5 m above the ground. It reaches its greatest height, 5 m, at a distance of 3 m from the nozzle. The graph shows the path.



a) Write an equation for the height of the water. Let x be the distance from the nozzle in metres, and y the height in metres.

b) Find the height of the water 2 m from the nozzle.

c) Find the height that the equation gives 8 m from the nozzle.

d) Which of your answers to b) and c) makes sense? Explain, using the words interpolation and extrapolation.

4. A ball is kicked from the ground. Two students describe its path. In both, x is the distance from where the ball was kicked, and y is the height of the ball, both in metres.

Student A writes: "The ball leaves the ground at 0 m and lands 30 m away. When it is 5 m from the start, it is 5 m high."

Student B makes a table of values.

x	y
10	8
15	9
20	8

a) Which form of equation suits Student A's description? Why? Write the equation.

b) Which form of equation suits Student B's table? Why? Write the equation.

c) Show that the two students describe the same path. Use one point.

Modelling with Both Forms — Hints

Stuck? Read the hint for that question, then go back and try again. Each hint points back to an idea from today's task cards. It does not give you the answer.

Question 1

Recall Card 1 (Question 1 on the discussion page): which point of the firework's path did we use first, and which other point did we know exactly? For b), recall Card 2 (Question 2 on the discussion page). For d), recall Card 3 (Question 3 on the discussion page).

Question 2

Recall Card 5 (Question 5 on the discussion page): where an arch meets the ground, or the water, gives two points you know. For c), recall Card 6 (Question 6 on the discussion page). For e), recall Card 3 (Question 3 on the discussion page): the axis of symmetry gives a matching point.

Question 3

Recall Card 1 (Question 1 on the discussion page) for the equation. Then recall Card 4 (Question 4 on the discussion page), where one prediction made sense and one did not.

Question 4

Recall Card 1 (Question 1 on the discussion page) and Card 5 (Question 5 on the discussion page): which information did each one give you? For the table, recall from the Analysing Vertex Form and Equations from Graphs summary how two points at the same height show the axis of symmetry.