Quadratic Relations and Finite Differences

Adapted from Principles of Mathematics 10, McGraw-Hill Ryerson, 2007.

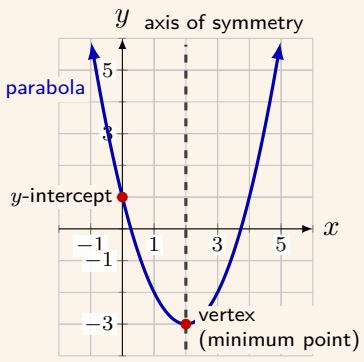
Read the Recall box, then try the questions. Show your difference columns. Plot on the grids printed here.

Recall: Quadratic Relations and Finite Differences

In a table whose x values are **evenly spaced**, subtract each y value from the next to get the **first differences**; subtract each first difference from the next to get the **second differences**.

Linear: the first differences are constant.
Quadratic: the first differences are not constant, but the second differences are.
Neither: neither column is constant.

The graph of a quadratic relation is a U-shaped curve called a **parabola**. It **opens up** or **opens down**. Its turning point is the **vertex**. It is symmetric about a vertical line through the vertex, the **axis of symmetry**. When it opens up, the y value at the vertex is the **minimum value**; when it opens down, the **maximum value**. The **y-intercept** is where it crosses the y-axis, the value of y when $x = 0$.



1. Complete the difference columns for each table. Then decide whether the relation is linear, quadratic, or neither, and say how you know.

a)

x	y	First differences	Second differences
−2	7		
−1	4		
0	1		
1	−2		
2	−5		

b)

x	y	First differences	Second differences
0	1		
1	5		
2	11		
3	19		
4	29		

_____ _____

_____ _____

c)

x	y	First differences	Second differences
1	38		
2	32		
3	22		
4	8		
5	−10		

d)

x	y	First differences	Second differences
0	2		
1	3		
2	10		
3	29		
4	66		

2. A relation can be described in words, as a table of values, as an equation, and as a graph. Here is one as an equation: $y = -x^2 + 6x - 5$. Describe it the other three ways.

x	y
0	
1	
2	
3	
4	
5	
6	

a) Complete the table of values.

b) Plot the points on the grid and join them with a smooth curve.

c) The parabola opens _____

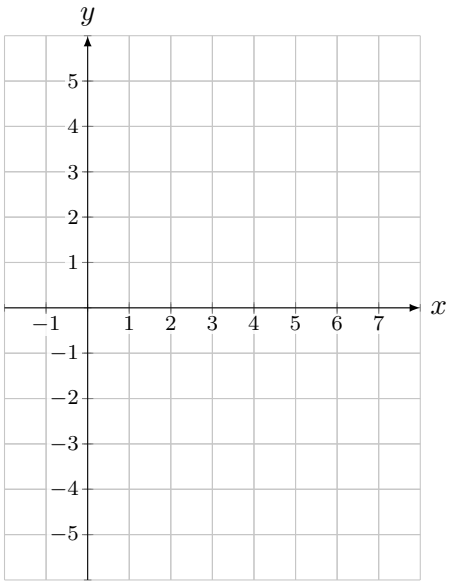
d) Label the vertex on your graph. Vertex: _____

e) Draw the axis of symmetry as a dashed line. Its equation: _____

f) The y-intercept: _____

g) Is the y value at the vertex a maximum or a minimum? _____

Its value: _____



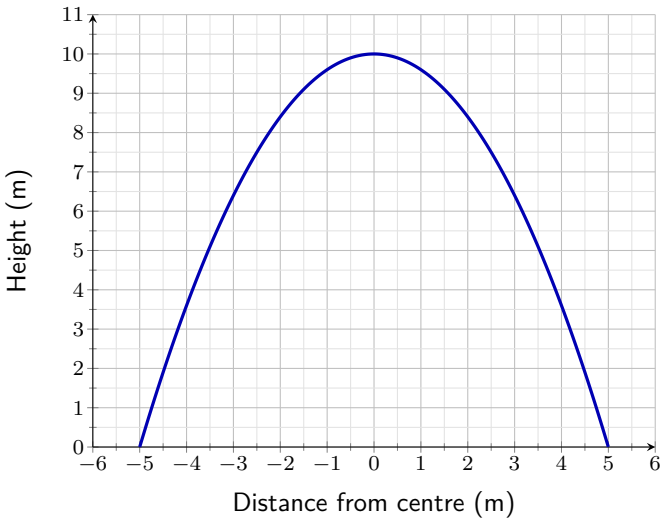
h) Now describe the relation in words.

3. A footbridge has an arched underside that follows $y = -0.4x^2 + 10$. Here x is the horizontal distance from the middle of the bridge and y is the height above the ground, both in metres. Answer parts a) to d) from the graph alone.

a) What shape is the arch? Use at least three words from the Recall box.

_____

_____



b) Height at the centre: _____

c) Height 3 m from the centre: _____

d) Width at the base: _____

e) Now use the equation to check your answer to part c).

_____

4. On Day 1 you plotted palm area against thumb length, and the points bent. The thumb lengths in my sample data were 5.3, 5.5, 6.0, 6.2, 6.4, 6.5, 6.7, 6.9, 7.1, 7.3, 7.5, and 7.8 cm.

a) Could you use finite differences on this data, as it is, to decide whether the relation is quadratic? Explain.

_____

_____

b) Describe what you could do instead to test whether the relation is close to quadratic.

_____

_____

c) Measured data almost never give second differences that are *exactly* equal. Why not?

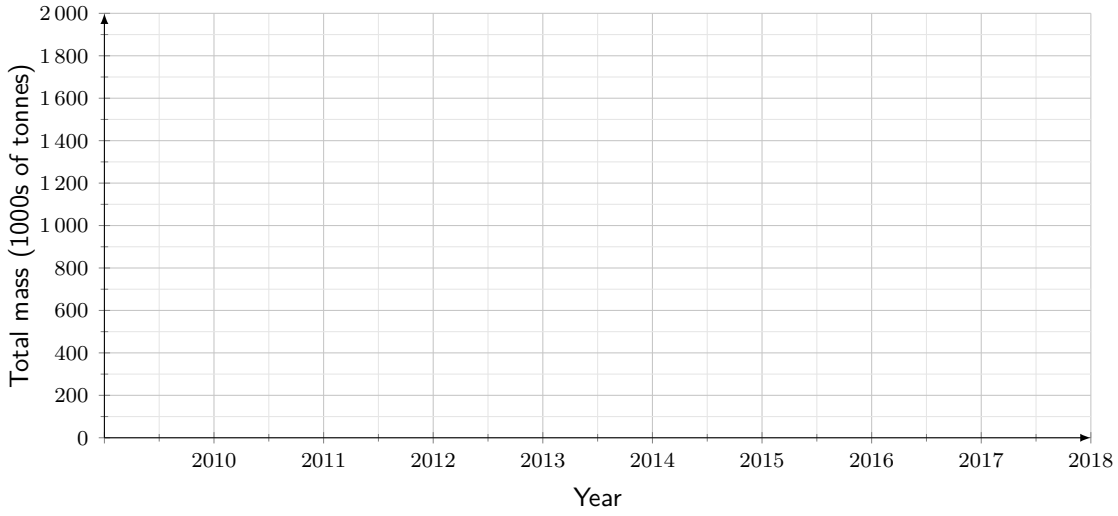
5. A new landfill began taking garbage in 2010. Each year more arrives; the second column records how much arrived during each year from 2010 to 2017.

Year	Garbage added (1000s of tonnes)	Total mass at end of year (1000s of tonnes)	First differences	Second differences
2010	150			
2011	172			
2012	196			
2013	219			
2014	241			
2015	265			
2016	288			
2017	310			

a) Complete the third column with a running total: how much garbage the landfill holds once each year is over.

b) Plot total mass against year on the grid below, and draw a curve of best fit.

c) Find the first and second differences of the *total* mass. Is the relation between total mass and year closer to linear or to quadratic? Justify your answer on the lines below.



Quadratic Relations and Finite Differences — Final Answers

Adapted from Principles of Mathematics 10, McGraw-Hill Ryerson, 2007.

Final answers only. Readings from a graph and curves drawn by eye will differ a little from mine, so each reading has a range of accepted answers.

Finite differences

1. a)

x	y	First differences	Second differences
-2	7		
		-3	
-1	4		0
		-3	
0	1		0
		-3	
1	-2		0
		-3	
2	-5		

Linear: the first differences are constant (-3).

b)

x	y	First differences	Second differences
0	1		
		4	
1	5		2
		6	
2	11		2
		8	
3	19		2
		10	
4	29		

Quadratic: the first differences change, but the second differences are constant (2).

c)

x	y	First differences	Second differences
1	38		
		-6	
2	32		-4
		-10	
3	22		-4
		-14	
4	8		-4
		-18	
5	-10		

Quadratic: the first differences change, but the second differences are constant (-4).

d)

x	y	First differences	Second differences
0	2		
		1	
1	3		6
		7	
2	10		12
		19	
3	29		18
		37	
4	66		

Neither: the first differences change, and so do the second differences.

2. A relation can be described in words, as a table of values, as an equation, and as a graph. Here is one as an equation: $y = -x^2 + 6x - 5$. Describe it the other three ways.

x	y
0	-5
1	0
2	3
3	4
4	3
5	0
6	-5

a) See the table.

b) See the grid.

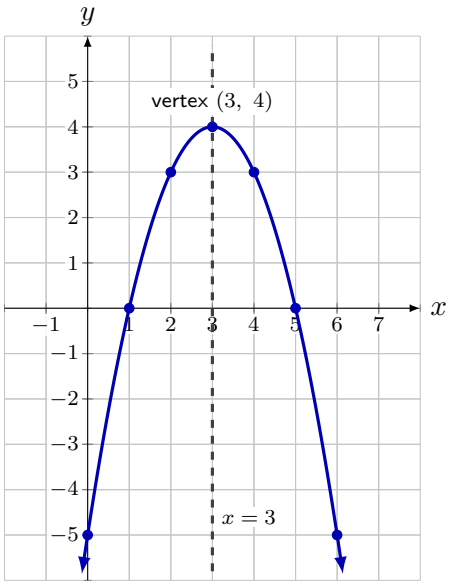
c) Opens down.

d) Vertex $(3,\ 4)$.

e) $x = 3$.

f) -5.

g) A maximum; the maximum value is 4.



h) A parabola that opens down, with its vertex, the maximum point, at $(3,\ 4)$, its axis of symmetry $x = 3$, and its y-intercept -5.

3. A footbridge has an arched underside that follows $y = -0.4x^2 + 10$. Here x is the horizontal distance from the middle of the bridge and y is the height above the ground, both in metres. Answer parts a) to d) from the graph alone.

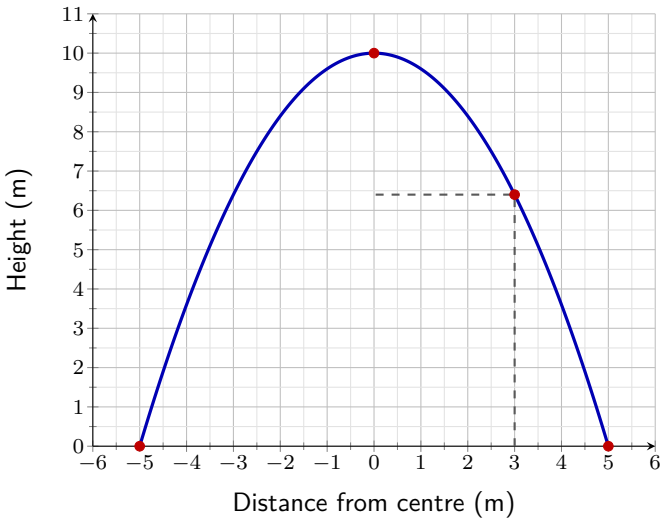
a) A parabola that opens down, with its vertex, the maximum point, at $(0,\ 10)$; it is symmetric about the axis of symmetry $x = 0$ (the y-axis).

b) 10 m (accept 9.8 m to 10.2 m).

c) About 6.4 m (accept 6 m to 6.8 m).

d) 10 m: it meets the ground at $x = -5$ and $x = 5$ (accept 9.6 m to 10.4 m).

e) $y = -0.4(3)^2 + 10 = -3.6 + 10 = 6.4$ m.



4. On Day 1 you plotted palm area against thumb length, and the points bent. The thumb lengths in my sample data were 5.3, 5.5, 6.0, 6.2, 6.4, 6.5, 6.7, 6.9, 7.1, 7.3, 7.5, and 7.8 cm.

a) No. Finite differences need evenly spaced x values, and these thumb lengths are not evenly spaced: the gaps are 0.2, 0.5, 0.2, 0.2, 0.1, ..., 0.3 cm. A bigger gap gives a bigger difference whatever the relation is.

b) Draw a curve of best fit, then read palm areas off the curve at evenly spaced thumb lengths (for example 5.5, 6, 6.5, 7, and 7.5 cm), and find the first and second differences of those readings. If the second differences are approximately constant and the first are not, the relation is close to quadratic.

c) Each measurement has some error, and each hand differs from the trend, so the points scatter about the curve. The best we can expect is second differences that are approximately constant.

5. A new landfill began taking garbage in 2010. Each year more arrives; the second column records how much arrived during each year from 2010 to 2017.

Year	Garbage added (1000s of tonnes)	Total mass at end of year (1000s of tonnes)	First differences	Second differences
2010	150	150		
			172	
2011	172	322		24
			196	
2012	196	518		23
			219	
2013	219	737		22
			241	
2014	241	978		24
			265	
2015	265	1243		23
			288	
2016	288	1531		22
			310	
2017	310	1841		

a) See the table.

b) See the grid.

c) Closer to quadratic. The first differences grow from 172 to 310, so it is not linear. The second differences (24, 23, 22, 24, 23, 22) are not exactly equal, but they are all close to 23, so they are approximately constant.

