Stretches, Reflections, and the Step Pattern

Adapted from Principles of Mathematics 10, McGraw-Hill Ryerson, 2007.

Read the Recall box, then try the questions. Sketch on the grids printed here.

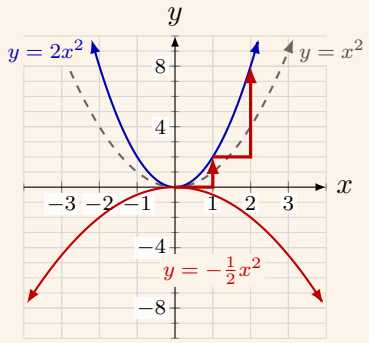
Recall: $y = ax^2$ and the Step Pattern

In $\boldsymbol{y = ax^2}$, every y-value of $y = x^2$ is multiplied by a.

- $a > 1$ or $a < -1$: a **vertical stretch**; the graph is narrower.
- $-1 < a < 1$ $(a \neq 0)$: a **vertical compression**; the graph is wider.
- $a < 0$: a **reflection in the x-axis**; the graph opens down.

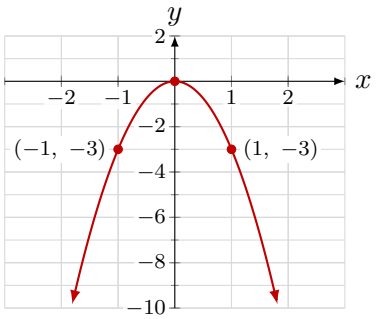
The vertex stays $(0,\ 0)$ and the axis of symmetry stays $x = 0$. In $y = ax^2 + k$, the vertex moves to $(0,\ k)$.

The step pattern: from the vertex, go over 1 and up a, over 1 and up $3a$, over 1 and up $5a$, ..., then mirror the steps on the other side. For $y = 2x^2$: up 2, then up 6, then up 10.



1. Each parabola has its vertex and some points marked. Write its equation, and show how you found a.

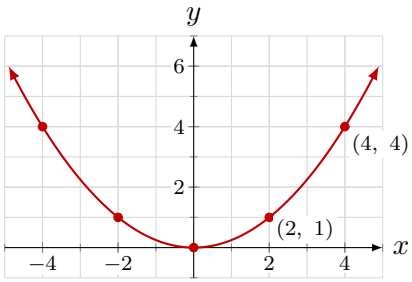
a)



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_____

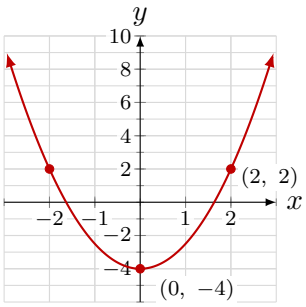
b)



_____

_____

c)



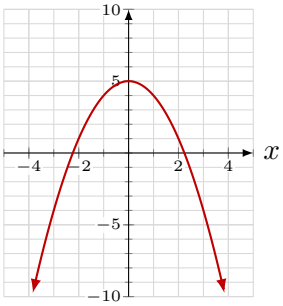
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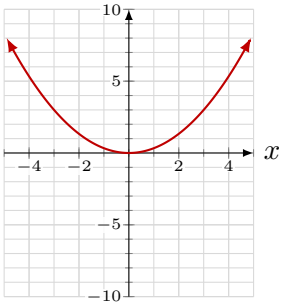
2. Match each equation with its graph. Then explain how you told graph A from graph C.

i) $y = 3x^2$ _____ ii) $y = \frac{1}{3}x^2$ _____ iii) $y = -x^2 + 5$ _____ iv) $y = -3x^2 + 5$ _____

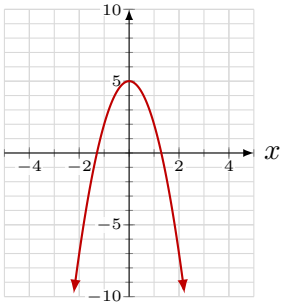
A



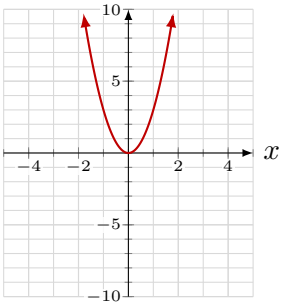
B



C



D



3. This half of a parabola is given as a table of values. The relation has the form $y = ax^2$.

x	y	First differences
0	0.0	
1	−0.4	
2	−1.6	
3	−3.6	
4	−6.4	

a) Fill in the first differences.

b) Use the step pattern to find a. Write the equation, and check it with one point.

c) In words, how does this graph compare with $y = x^2$?

d) Without calculating, give y when $x = -3$. Why?

4. Equation to words. In a sentence, describe how each graph compares with $y = x^2$. Say which way it opens and give its vertex.

a) $y = 5x^2$

b) $y = -0.2x^2$

c) $y = \frac{2}{3}x^2 + 1$

Words to equation. Write the equation of each parabola.

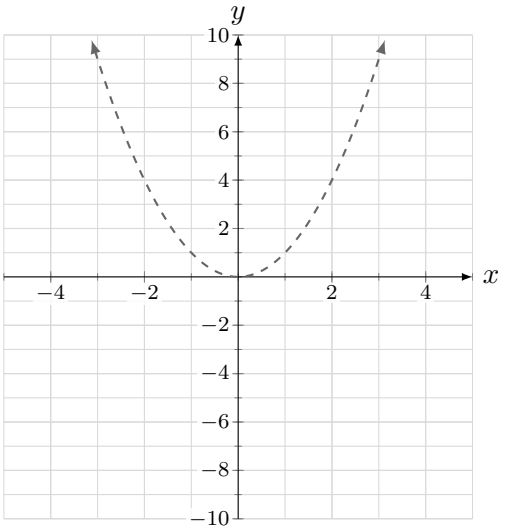
d) $y = x^2$ stretched vertically by a factor of 6.

e) $y = x^2$ compressed vertically by a factor of $\frac{1}{4}$ and reflected in the x-axis.

f) $y = x^2$ reflected in the x-axis, stretched vertically by a factor of 3, and then translated 2 units down.

5. Without making a table, use the step pattern to sketch each relation on the grid with $y = x^2$ (dashed). Plot only points that fit on the grid. State each vertex and the steps you used.

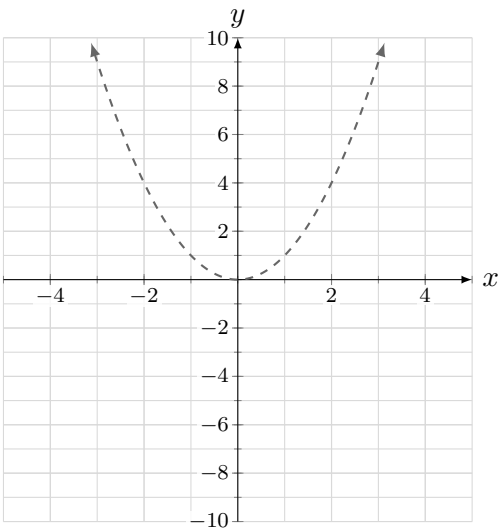
a) $y = 3x^2 - 5$



Vertex _____

Steps from the vertex _____

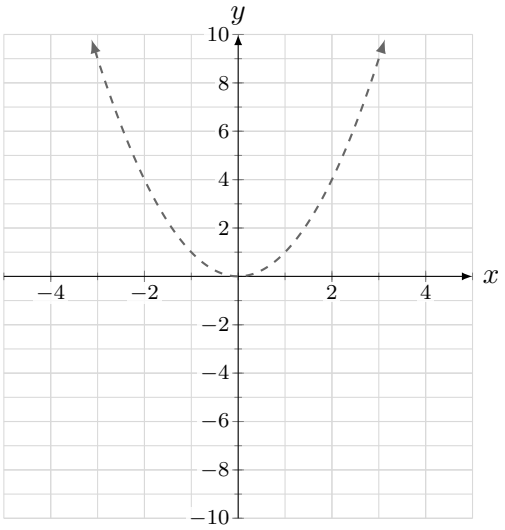
b) $y = -\frac{1}{2}x^2$



Vertex _____

Steps from the vertex _____

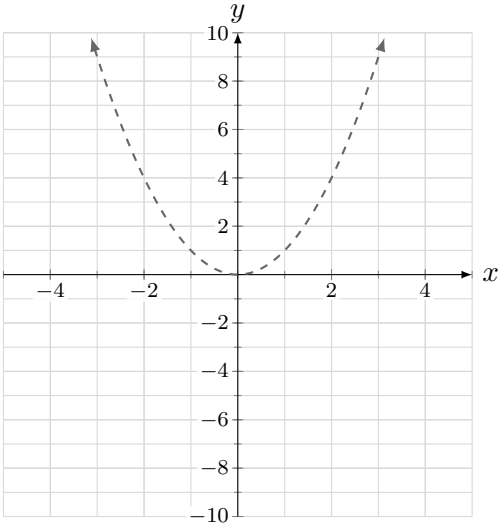
c) $y = \frac{1}{2}x^2 - 6$



Vertex _____

Steps from the vertex _____

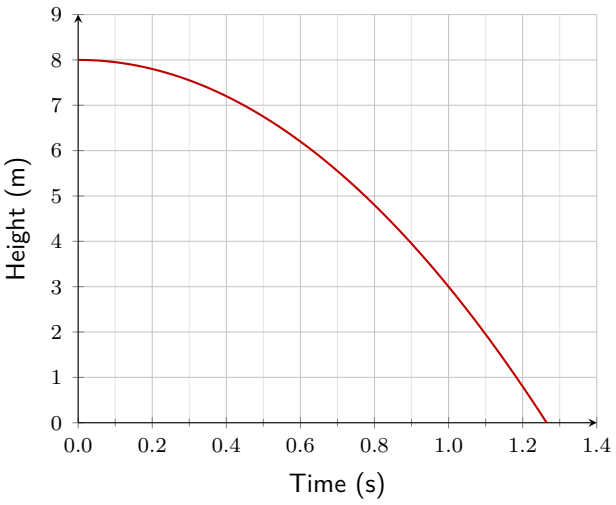
d) $y = -2x^2 + 7$



Vertex _____

Steps from the vertex _____

6. A ball is dropped from a window. Its height above the ground, y metres, after x seconds is modelled by $y = -5x^2 + 8$.



a) From what height was it dropped? Use the graph, then the equation.

b) From the graph, about when does the ball reach the ground?

c) Use the equation to find the height after 1 second. Mark it on the graph.

d) Why is only part of a parabola shown?

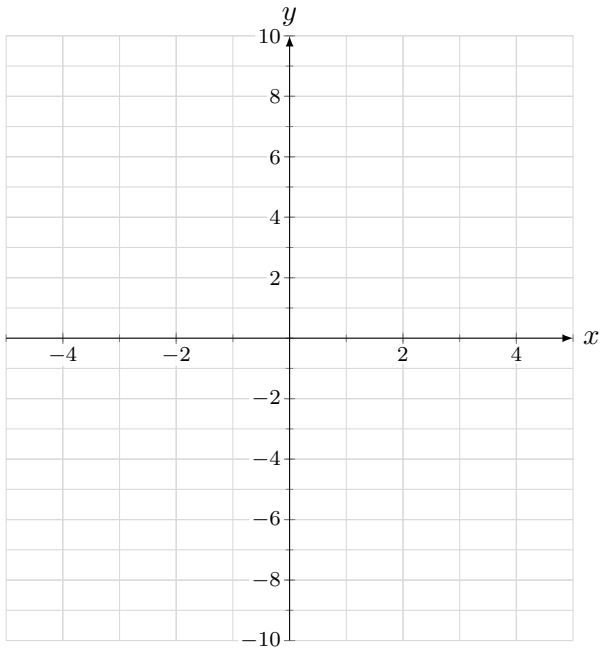
e) How would the equation change if the ball were dropped from 12 m?

7. A relation can be given in words, as a table of values, as an equation, and as a graph. This parabola is given as an equation: $y = -2x^2 + 3$. Give it the other three ways.

a) As a table of values

x	y
-2	
-1	
0	
1	
2	

b) As a graph: plot your table, and check it with the step pattern.



c) In words: how is it made from $y = x^2$? Which way does it open? Give its vertex, its axis of symmetry, and its maximum or minimum value.

Stretches, Reflections, and the Step Pattern — Hints

Stuck? Read the hint for that question, then go back and try again. Each hint points back to an idea from today's discussion. It does not give you the answer.

Question 1

Recall Card 7 (Question 7 on the discussion page): we found the coefficient a more than one way, by the step pattern and by substitution. When you cannot read the first step from the vertex exactly, substitute a marked point into the equation instead. Recall what the direction a parabola opens tells you about a, and for c), recall where the vertex of $y = ax^2 + k$ sits.

Question 2

Recall from the summary that the size of the coefficient a sets the width, and the sign of a sets the direction. Sort the graphs by which way they open and where each vertex sits, then compare widths. To explain how you told A from C, recall the step pattern: what does the first step from the vertex tell you about a?

Question 3

Each first difference is the lower value minus the one above it, so watch the signs. Recall Card 5 (Question 5 on the discussion page): from the vertex, the first differences of $y = ax^2$ are a, $3a$, $5a$, and so on. For c), recall what the size and the sign of the coefficient a each tell you, and for d), recall what the axis of symmetry tells you.

Question 4

Recall the summary's three cases for the coefficient a: its size decides between a vertical stretch and a vertical compression, and its sign decides whether there is a reflection in the x-axis. Then recall what k does in $y = ax^2 + k$, and where it puts the vertex. For d) to f), work the other way: read the size of a, its sign, and k from the words.

Question 5

Recall Card 6 (Question 6 on the discussion page): we used only the step pattern, with no table, and stopped when the next step would leave the grid. With a translation, start at the vertex of $y = ax^2 + k$, and recall which way the steps go when the coefficient a is negative.

Question 6

Think about what x and y each measure, and which points on the graph show the moment the ball is dropped and the moment it lands. For c), recall the order of operations in $y = ax^2 + k$: square first. For d), think about which times make sense for a falling ball. For e), recall what k does to the graph of $y = ax^2 + k$ and where it puts the vertex.

Question 7

Recall the summary's $y = 2x^2 - 3$ four ways, and use it as a model. For the table, square x first, then multiply by the coefficient a, then add k; recall that $-2x^2$ means $-2(x^2)$. Check your graph with the step pattern from the vertex $(0, k)$, and for the words, recall what the size of a, the sign of a, and k each do.