Vertical Translations of the Parabola

Adapted from Principles of Mathematics 10, McGraw-Hill Ryerson, 2007.

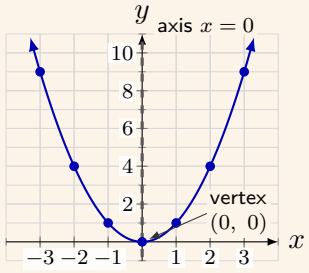
Read the Recall box, then try the questions. Sketch on the grids printed here.

Recall: The Parent Parabola and $y = x^2 + k$

The graph of $y = x^2$ is a **parabola**. Its lowest point, the **vertex**, is $(0,\ 0)$. It is symmetric about the **axis of symmetry**, the vertical line $x = 0$: two x-values the same distance from 0 give the same y-value.

The graph of $\boldsymbol{y = x^2 + k}$ is the graph of $y = x^2$ **translated vertically** k units: up when $k > 0$, down when $k < 0$. Every y-value changes by k, so the vertex moves to $(0,\ k)$, while the shape and the axis of symmetry stay the same.

x	y
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9



1.

x	y	First differences	Second differences
-3			
-2			
-1			
0			
1			
2			
3			

Each difference is the lower value minus the one above it.

Complete the table for $y = x^2 - 1$, with its first and second differences. Then state:

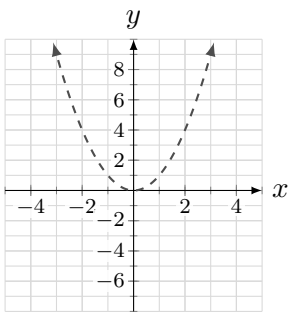
Vertex _____

Axis of symmetry _____

y-intercept _____

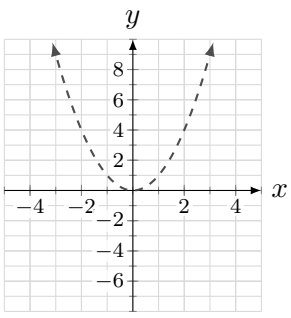
2. Without making a table, sketch each relation on the grid with $y = x^2$ (dashed). State each vertex.

a) $y = x^2 + 4$



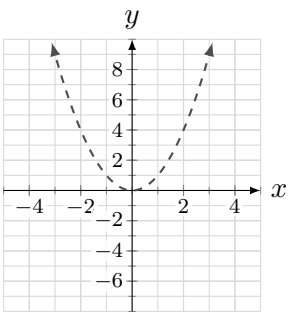
Vertex _____

b) $y = x^2 - 3$



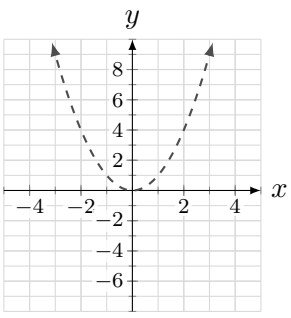
Vertex _____

c) $y = x^2 + 1$



Vertex _____

d) $y = x^2 - 7$



Vertex _____

3. Describe each relation in a sentence. Say which way the parabola opens, and give its vertex, its axis of symmetry, its y-intercept, and its maximum or minimum value.

a) $y = x^2 + 8$

b) $y = x^2 - 5$

c) $y = x^2 + 0.5$

d) $y = x^2 - 12$

4. Write the equation of each parabola.

a) The graph of $y = x^2$ translated 6 units up.

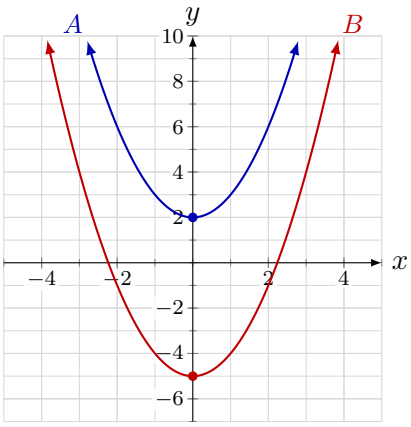
b) The graph of $y = x^2$ translated 9 units down.

c) Vertex $(0,\ 2)$, and the same shape as $y = x^2$.

d) The same shape as $y = x^2$, with its vertex on the y-axis, passing through $(0,\ -3)$.

e) The same shape as $y = x^2$, with its vertex on the y-axis, passing through $(2,\ 0)$. *Hint: what would $y = x^2$ give at $x = 2$?*

5. Parabolas A and B each have the same shape as $y = x^2$.



a) Write the equation of parabola A.

b) Write the equation of parabola B.

c) How far apart are the two parabolas, measured straight up and down? Is the distance the same for every x? Explain.

6. Consider the relations $y = x^2 - 16$ and $y = x^2 + 16$.

a) Where does the graph of $y = x^2 - 16$ meet the x-axis? Make a table, or reason it out. Show how you know.

b) Explain why the graph of $y = x^2 + 16$ never meets the x-axis.

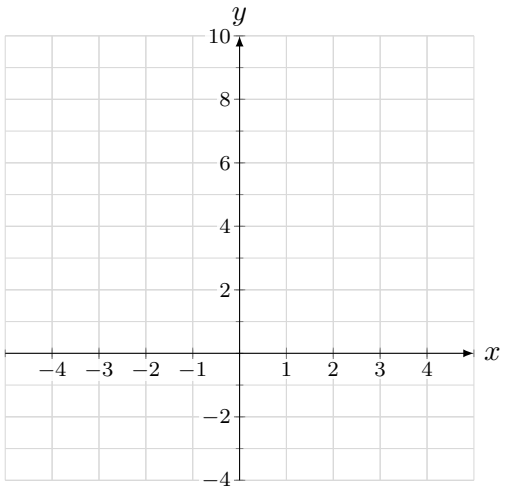
7. A relation can be given in words, as a table of values, as an equation, and as a graph. This parabola is given as a table of values. Give it the other three ways.

x	y
-3	7
-2	2
-1	-1
0	-2
1	-1
2	2
3	7

a) In words: which way does it open, where is its vertex, what is its axis of symmetry, and how does it compare with $y = x^2$?

b) As an equation. Check your equation with one point from the table.

c) As a graph: plot the points and sketch the parabola on the grid.



Vertical Translations of the Parabola — Final Answers

Adapted from Principles of Mathematics 10, McGraw-Hill Ryerson, 2007.

Final answers only. Sketches are drawn on the grids in blue, beside the printed $y = x^2$.

The parent parabola and $y = x^2 + k$

1.

x	y	First differences	Second differences
-3	8		
		$3 - 8 = -5$	
-2	3		$-3 - (-5) = 2$
		$0 - 3 = -3$	
-1	0		$-1 - (-3) = 2$
		$-1 - 0 = -1$	
0	-1		$1 - (-1) = 2$
		$0 - (-1) = 1$	
1	0		$3 - 1 = 2$
		$3 - 0 = 3$	
2	3		$5 - 3 = 2$
		$8 - 3 = 5$	
3	8		

Each difference is the lower value minus the one above it.

The table for $y = x^2 - 1$.

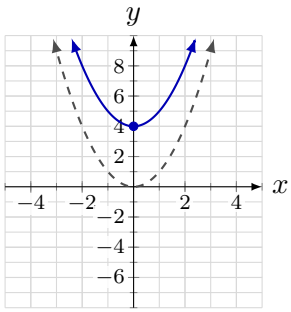
Vertex $(0,\ -1)$

Axis of symmetry $x = 0$

y-intercept -1, at $(0,\ -1)$

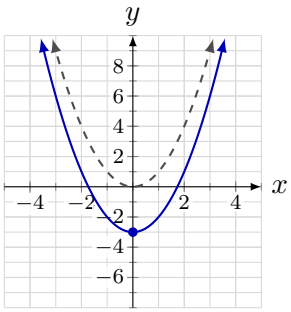
2. Each sketch is $y = x^2$ moved up or down; the vertex is the lowest point.

a) $y = x^2 + 4$



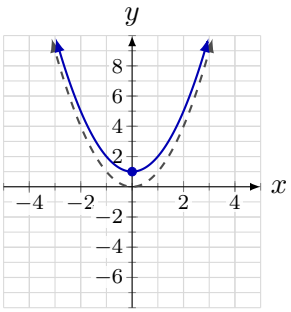
Vertex $(0,\ 4)$

b) $y = x^2 - 3$



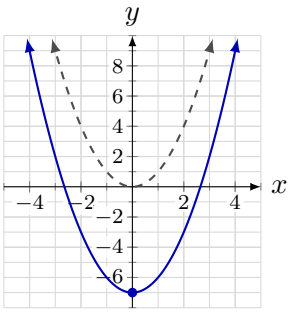
Vertex $(0,\ -3)$

c) $y = x^2 + 1$



Vertex $(0,\ 1)$

d) $y = x^2 - 7$



Vertex $(0,\ -7)$

3. a) $y = x^2 + 8$ opens upward, with vertex $(0,\ 8)$, axis of symmetry $x = 0$, y-intercept 8, and a minimum value of 8.

b) $y = x^2 - 5$ opens upward, with vertex $(0,\ -5)$, axis of symmetry $x = 0$, y-intercept -5, and a minimum value of -5.

c) $y = x^2 + 0.5$ opens upward, with vertex $(0,\ 0.5)$, axis of symmetry $x = 0$, y-intercept 0.5, and a minimum value of 0.5.

d) $y = x^2 - 12$ opens upward, with vertex $(0,\ -12)$, axis of symmetry $x = 0$, y-intercept -12, and a minimum value of -12.

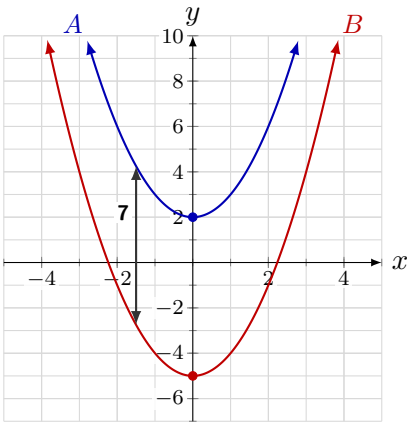
None has a maximum value: each opens upward, so y grows without limit.

4. a) $y = x^2 + 6$ b) $y = x^2 - 9$

c) $y = x^2 + 2$ d) $y = x^2 - 3$

e) $y = x^2 - 4$. At $x = 2$, $y = x^2$ gives 4; the point is at 0, which is 4 lower, so $k = -4$. Check: $2^2 - 4 = 0$.

5. Parabolas A and B each have the same shape as $y = x^2$.



a) A: $y = x^2 + 2$ (vertex $(0,\ 2)$)

b) B: $y = x^2 - 5$ (vertex $(0,\ -5)$)

c) 7 units, at every x: for any x, $(x^2 + 2) - (x^2 - 5) = 7$. For example, at $x = 3$: $11 - 4 = 7$.

6. a) At $(-4,\ 0)$ and $(4,\ 0)$. On the x-axis, $y = 0$, so $x^2 = 16$ and $x = 4$ or $x = -4$. Check: $4^2 - 16 = 0$ and $(-4)^2 - 16 = 0$.

b) x^2 is never negative, so $x^2 + 16$ is always at least 16. Its vertex $(0,\ 16)$ is its lowest point, 16 units above the x-axis, and the parabola opens upward, so y is never 0.

7. A relation can be given in words, as a table of values, as an equation, and as a graph. The parabola given as a table of values:

x	y
-3	7
-2	2
-1	-1
0	-2
1	-1
2	2
3	7

a) In words: it opens upward, its vertex is $(0,\ -2)$, and its axis of symmetry is $x = 0$. Every y-value is 2 less than for $y = x^2$ (9 to 7, 4 to 2, 1 to -1, 0 to -2), so it is the graph of $y = x^2$ translated 2 units down.

b) As an equation: $y = x^2 - 2$. Check $(3,\ 7)$: $3^2 - 2 = 7$; check $(-1,\ -1)$: $(-1)^2 - 2 = -1$.

c) As a graph:

