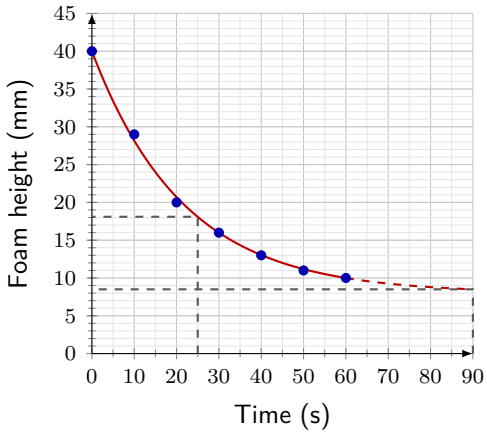
Reviewing Quadratic Relations — Final Answers

Final answers only. Readings from a graph and curves drawn by eye will differ a little from mine, so each reading has a range of accepted answers.

Relations and models

1. Foam height: accept readings within the ranges given; a curve drawn by eye will differ a little from mine.

Time (s)	Foam (mm)
0	40
10	29
20	20
30	16
40	13
50	11
60	10



a) Time is independent; foam height is dependent.

b) See the graph: a smooth curve that falls quickly, then levels off.

c) Negative, strong, and non-linear: the foam falls quickly at first, then more and more slowly.

d) 25 s: about 18 mm (accept 17 to 20), an interpolation. 90 s: about 8 or 9 mm (accept 6 to 10), an extrapolation.

e) 25 s: it lies inside the data. At 90 s I assume the curve keeps levelling off as it did; nothing was measured there.

2. a) interpolation b) extrapolation c) extrapolation
before the data begin

d) Week 20: it is farthest beyond the data, and a puppy's growth slows as it becomes an adult, so the trend may not carry on. (Week 0 is also fair with a reason: it lies beyond the data too.)

Finite differences

3. a)

x	y	First differences	Second differences
1	5		
		$1-5=-4$	
2	1		$-4-(-4)=0$
		$-3-1=-4$	
3	−3		$-4-(-4)=0$
		$-7-(-3)=-4$	
4	−7		$-4-(-4)=0$
		$-11-(-7)=-4$	
5	−11		

Each difference is the lower value minus the one above it.

Linear: the first differences are constant (−4).

b)

x	y	First differences	Second differences
0	1		
		2	
1	3		4
		6	
2	9		12
		18	
3	27		36
		54	
4	81		

Neither: neither column is constant. Each y is 3 times the one before.

c)

x	d	First differences	Second differences
0	0.0		
		1.2	
1	1.2		2.5
		3.7	
2	4.9		2.1
		5.8	
3	10.7		2.6
		8.4	
4	19.1		

Close to quadratic: measured data, with second differences roughly constant (about 2.4).

4.

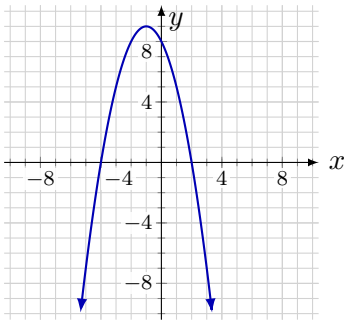
x	y
1	-1
2	8
3	23
5	71
6	104

a) The x values are not evenly spaced: x jumps from 3 to 5. The difference 48 covers two steps, so the rule does not apply.

b) First differences 9, 15, so next 21 and 27: $y = 23 + 21 = 44$, and $44 + 27 = 71$ checks. Completed: -1, 8, 23, 44, 71, 104; first differences 9, 15, 21, 27, 33; second differences all 6.

Key features and the four ways to describe a relation

5.

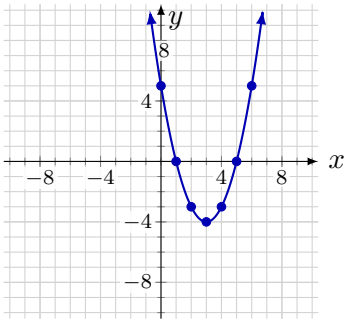


Opens: down
Vertex: $(-1, 9)$
Axis: $x = -1$
Max or min value: maximum 9

Zeros: -4 and 2
y-intercept: 8
x may take: any number
y may take: $y \leq 9$

6. $y = x^2 - 6x + 5$

x	y
0	5
1	0
2	-3
3	-4
4	-3
5	0
6	5



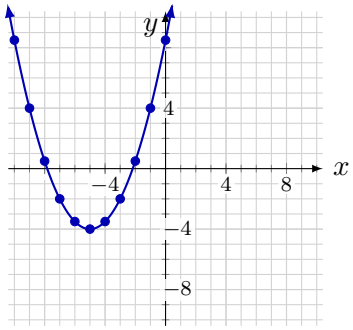
In words: a parabola that opens up, with its vertex at $(3, -4)$, its axis of symmetry $x = 3$, its minimum value -4, its zeros 1 and 5, and its y-intercept 5.

Transformations and the step pattern

7. The vertex and the step points are marked.

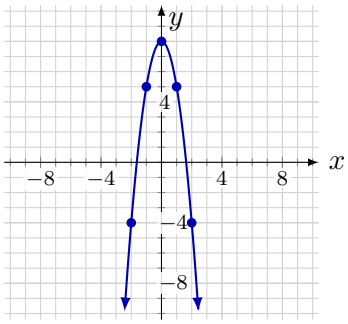
a) $y = \frac{1}{2}(x+5)^2 - 4$

vertex $(-5, -4)$; steps up $\frac{1}{2}$, $\frac{3}{2}$, $\frac{5}{2}$, $\frac{7}{2}$, $\frac{9}{2}$



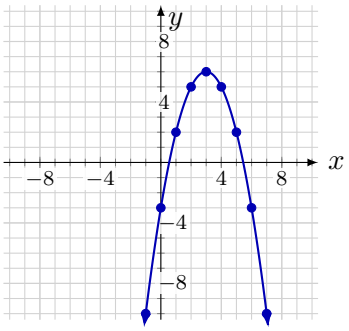
b) $y = -3x^2 + 8$

vertex $(0, 8)$; steps down 3, 9



c) $y = -(x-3)^2 + 6$

vertex $(3, 6)$; steps down 1, 3, 5, 7



8. a) vertical stretch by a factor of 4, 6 units left, and 1 unit down; vertex $(-6, -1)$, axis $x = -6$

b) reflection in the x-axis, vertical compression by a factor of $\frac{1}{3}$, 5 units right, and 2 units up; vertex $(5, 2)$, axis $x = 5$

c) $y = -3(x+2)^2 + 5$

d) $y = \frac{1}{4}(x-7)^2$

Equation from a graph

9. a) $y = -3x^2 + 4$

b) $y = 2(x+4)^2 - 2$

c) $y = -\frac{1}{3}(x-1)^2 + 6$

10. a) $y = \frac{1}{2}(x+6)^2 + 1$

$9 = 16a + 1$

b) $y = -4(x-3)^2 - 2$

c) $y = 3(x-2)^2 - 3$

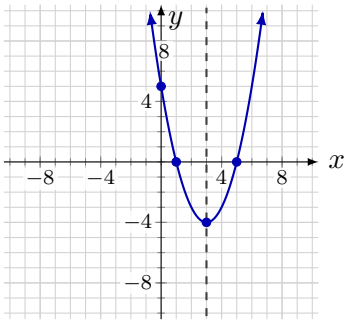
$9 = 4a - 3$

Intercepts form

11. The zeros, the vertex, and the y-intercept are marked; the axis is dashed.

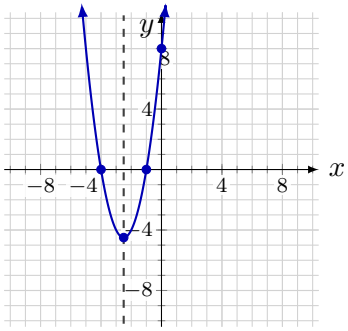
a) $y = (x-1)(x-5)$

zeros 1, 5; axis $x = 3$; vertex $(3, -4)$; y-int. 5



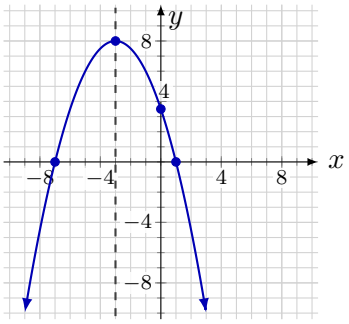
b) $y = 2(x+4)(x+1)$

zeros -4, -1; axis $x = -2.5$; vertex $(-2.5, -4.5)$; y-int. 8



c) $y = -\frac{1}{2}(x-1)(x+7)$

zeros 1, -7; axis $x = -3$; vertex $(-3, 8)$; y-int. 3.5



12. a) $y = -\frac{1}{2}(x+3)(x-5)$ b) $y = x(x+6)$ c) $y = 2(x-1)(x-4)$

13. a) $y = (x-4)(x-4)$: one zero, 4 (the vertex $(4, 0)$ is on the x-axis).

b) Cannot: it opens up with minimum value 3, so it never meets the x-axis and has no zeros.

c) $y = -(x+3)(x+1)$: it opens down with maximum value 1, so it has two zeros, -3 and -1.

Modelling with quadratic relations

14. a) $y = 0.2(x-4)^2 + 0.2$ $(3.4 = 16a + 0.2)$

b) 8 m: the right edge mirrors the left in the axis $x = 4$.

c) 2 m; also at 7 m across.

d) $h = 4$: the lowest point is 4 m from the left edge.
$k = 0.2$: the lowest point is 0.2 m above the ground.

15. a) $y = -0.024x(x-50)$ $(15 = a(25)(-25))$

b) 9.6 m; also 40 m away.

c) Yes: at 6 m the ball is about 6.3 m high.

d) -6.6 m: the ball has landed at 50 m, so the model holds only for $0 \le x \le 50$.

16. a) \$3200; interpolation.

b) $-$\$2200, a loss; extrapolation.

c) \$9; \$5000 (the vertex)

d) a): \$6 lies inside the prices tried; \$15 lies beyond them.

Negative and zero exponents

17. a) 1 b) $\frac{1}{64}$ c) $-\frac{1}{8}$ d) -1
e) 9 f) $\frac{5}{2}$ g) $\frac{1}{10\,000}$, or 0.0001 h) $\frac{7}{2}$, or $3\frac{1}{2}$

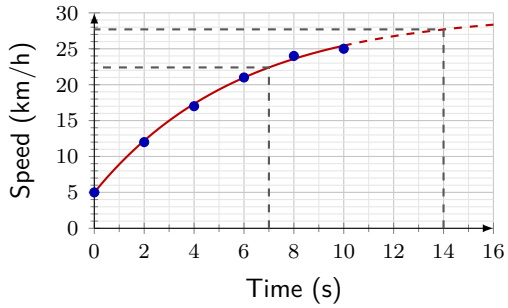
18. a) $810\,\text{m}^2$: the area on June 1.
b) 3 weeks before June 1; $810 \times \frac{1}{27} = 30\,\text{m}^2$.
c) 4 weeks ($810 \div 81 = 10$, so $t = -4$)
d) No: going back a week divides by 3, and dividing a positive number by 3 always leaves a positive number.

19. a) He multiplied the base by the exponent instead of using it as a power, and he made the answer negative. A negative exponent means one over the power: $5^{-3} = \frac{1}{5^3} = \frac{1}{125}$, which is positive.
b) Pattern: $4^3 = 64$, $4^2 = 16$, $4^1 = 4$; each step divides by 4, so $4^0 = 4 \div 4 = 1$. Or the law: $4^2 \div 4^2 = 4^{2-2} = 4^0$, and $16 \div 16 = 1$.

Mixed review

Readings from a graph will differ a little from mine, so each reading has a range of accepted answers.

20.



a) Positive, strong, and non-linear: the speed rises quickly, then levels off.
b) 7 s: about 22 km/h (accept 21 to 23.5). 14 s: about 28 km/h (accept 26.5 to 29).
c) 7 s is an interpolation (inside 0 to 10 s); 14 s is an extrapolation. Trust 7 s: there are data on both sides of it.

21. a) $y = -0.02(x - 10)^2 + 3.2$ b) 1.92 m c) 2 m from the thrower
$1.2 = 100a + 3.2$ mirror of 18 in $x = 10$

22.

x	y
0	30
2	26
4	18
6	6
8	−10

First differences -4, -8, -12, -16; second differences -4, -4, -4. Quadratic: the first differences change, but the second differences are constant. The x values step evenly by 2, so the rule applies.

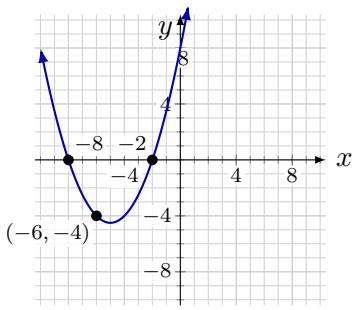
23. a) $\frac{1}{32}$ b) 1 c) 64 d) $\frac{9}{8}$

24. The bracket $x + 3$ is $(x - (-3))$, so $h = -3$: the graph moves 3 units *left*. The vertex is $(-3, -5)$, where the bracket is zero.

25. $y = -(x - 2)(x - 8)$ ($-16 = a(-2)(-8) = 16a$); vertex $(5, 9)$.

26. 8^{-2}: $8^{-2} = \frac{1}{64}$ and $2^{-8} = \frac{1}{256}$, and one sixty-fourth is more than one two hundred fifty-sixth.

27.



$y = \frac{1}{2}(x+8)(x+2)$

$-4 = a(2)(-4) = -8a$; vertex $(-5, -4.5)$, y-intercept 8

28.

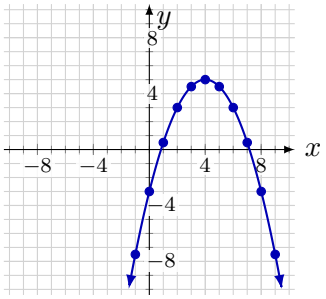
x	y
0	4
1	7
2	10
3	13
4	16

Constant first differences mean the relation is linear ($y = 3x+4$). A quadratic relation has first differences that are *not* constant and second differences that are constant (and not 0).

29. The vertex, the step points, and the y-intercept are marked.

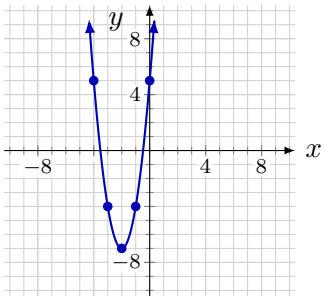
a) $y = -\frac{1}{2}(x-4)^2 + 5$

vertex $(4, 5)$; steps down $\frac{1}{2}$, $\frac{3}{2}$, $\frac{5}{2}$, $\frac{7}{2}$, $\frac{9}{2}$; y-int. -3



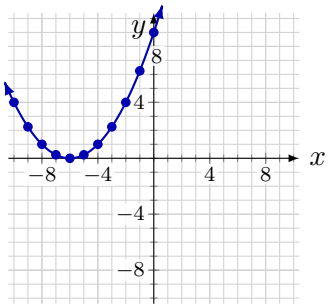
b) $y = 3(x+2)^2 - 7$

vertex $(-2, -7)$; steps up 3, 9; y-int. 5



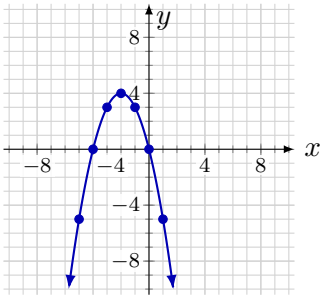
c) $y = \frac{1}{4}(x+6)^2$

vertex $(-6, 0)$; steps up $\frac{1}{4}$, $\frac{3}{4}$, $\frac{5}{4}$, $\frac{7}{4}$, $\frac{9}{4}$, $\frac{11}{4}$; y-int. 9



30. Vertex form $y = -(x+2)^2 + 4$; intercepts form $y = -x(x+4)$, zeros -4 and 0.

x	y
−5	−5
−4	0
−3	3
−2	4
−1	3
0	0
1	−5



31. $y = -0.15x(x-10)$ $(2.4 = a(2)(-8) = -16a)$; greatest height 3.75 m, 5 m from the sprinkler.

32. $y = -\frac{1}{2}(x-3)^2 + 6$ $(4 = -\frac{1}{2}(2)^2 + k$, so $k = 6)$

33. -6. From the vertex the steps are a, $3a$, $5a$, ..., so the first differences grow by $2a = -6$ each time.