Reviewing Quadratic Relations: Days 1 to 5 — Final Answers

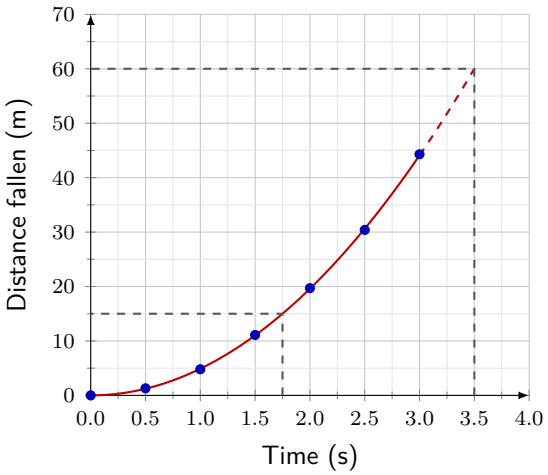
Adapted from Principles of Mathematics 10, McGraw-Hill Ryerson, 2007.

Final answers only. Readings from a graph and curves drawn by eye will differ a little from mine, so each reading has a range of accepted answers.

Relations and models

1. Readings: accept the ranges given; a curve drawn by eye will differ a little from mine.

Time (s)	Fallen (m)
0.0	0.0
0.5	1.3
1.0	4.8
1.5	11.1
2.0	19.7
2.5	30.4
3.0	44.3



a) Time is independent; the distance fallen is dependent.

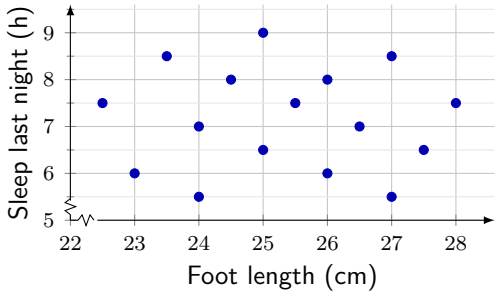
b) See the graph: a smooth curve that gets steeper and steeper.

c) Positive, strong, and non-linear: the stone falls farther in each half second than in the one before.

d) 1.75 s: about 15 m (accept 13 to 17), an interpolation. 3.5 s: about 60 m (accept 53 to 66), an extrapolation.

e) 1.75 s: it lies inside the data. At 3.5 s the stone is already in the water and no longer falls the same way, so the trend cannot carry on.

2. a) The temperature. Nobody controls it, but the café predicts its sales *from* it: the number of cold drinks sold depends on the temperature.



b) No clear direction, and weak: the points form a wide cloud with no trend. Long and short feet both have long and short nights of sleep.

c) No. A line through the cloud would be nearly flat and would predict almost nothing. The data show no relation to model.

Quadratic relations and finite differences

3. a)

x	y
0	1
1	4
2	5
3	4
4	1

First differences: $4-1=3$, $5-4=1$, $4-5=-1$, $1-4=-3$

Second differences: $1-3=-2$, $-1-1=-2$, $-3-(-1)=-2$

Each difference is the lower value minus the one above it.

Quadratic: the first differences change, but the second differences are constant (-2).

b)

x	y	First differences	Second differences
-2	7		
		-3	
-1	4		0
		-3	
0	1		0
		-3	
1	-2		0
		-3	
2	-5		

Linear: the first differences are constant (-3).

c)

x	y	First differences	Second differences
0	1		
		2	
1	3		2
		4	
2	7		4
		8	
3	15		8
		16	
4	31		

Neither: neither column is constant.

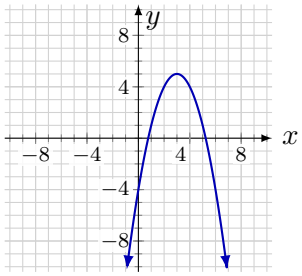
4.

x	y	First differences	Second differences
0	3		
		1	
1	4		1
		2	
2	6		2
		4	
3	10		4
		8	
4	18		

a) First differences 1, 2, 4, 8; second differences 1, 2, 4.

b) A curve only shows that the relation is not linear: many relations curve. A quadratic relation needs constant second differences, and these (1, 2, 4) are not constant. The relation is **neither**.

5. $y = -x^2 + 6x - 4$



a)

Opens: down
Vertex: (3, 5)
Axis of symmetry: $x = 3$
Max or min value: maximum 5
y-intercept: -4

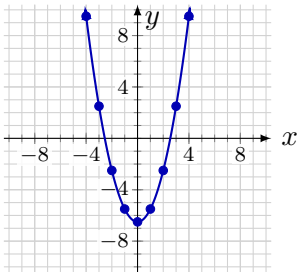
b) At $x = 0$: $y = -0 + 0 - 4 = -4$. Every term with an x becomes 0, leaving the constant term, -4.

The parent parabola and vertical translations

6. The vertex and the step points are marked.

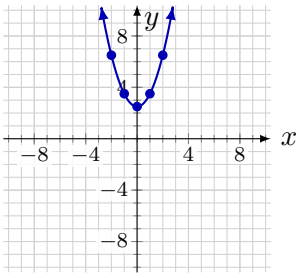
a) $y = x^2 - 6.5$

vertex $(0, -6.5)$; through $(\pm 1, -5.5)$, $(\pm 2, -2.5)$, $(\pm 3, 2.5)$, $(\pm 4, 9.5)$



b) $y = x^2 + 2.5$

vertex $(0, 2.5)$; through $(\pm 1, 3.5)$, $(\pm 2, 6.5)$



7. The completed table:

	Equation	In words	Vertex	Times it crosses the x-axis
a)	$y = x^2 + 9$	$y = x^2$ moved 9 units up	$(0,\ 9)$	0: it never meets the x-axis
b)	$y = x^2 - 14$	$y = x^2$ moved 14 units down	$(0,\ -14)$	2
c)	$y = x^2 - 3.5$	$y = x^2$ moved 3.5 units down	$(0,\ -3.5)$	2
d)	$y = x^2$	$y = x^2$, not moved ($k = 0$)	$(0,\ 0)$	1: it only touches the x-axis

8. $3^2 - 14 = -5$ is below the x-axis and $4^2 - 14 = 2$ is above it, so one crossing lies between 3 and 4. By symmetry, the other lies between -4 and -3.

Stretches, compressions, reflections, and the step pattern

9. a) stretch by a factor of 7;
up 7, 21, 35

b) compression by a factor of $\frac{1}{5}$ and a reflection;
down $\frac{1}{5}$, $\frac{3}{5}$, 1

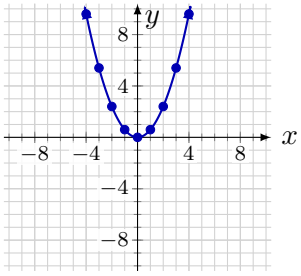
c) stretch by a factor of 6 and a reflection;
down 6, 18, 30

d) compression by a factor of 0.3;
up 0.3, 0.9, 1.5

10. The vertex and the step points are marked.

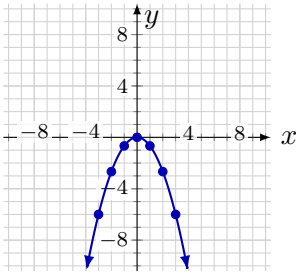
a) $y = 0.6x^2$

steps up 0.6, 1.8, 3.0, 4.2: $(\pm1, 0.6)$, $(\pm2, 2.4)$, $(\pm3, 5.4)$, $(\pm4, 9.6)$



b) $y = -\frac{2}{3}x^2$

steps down $\frac{2}{3}$, 2, $\frac{10}{3}$: $(\pm1, -\frac{2}{3})$, $(\pm2, -\frac{8}{3})$, $(\pm3, -6)$



11. a) Up 0.7, 2.1, 3.5, 4.9 (that is $1a$, $3a$, $5a$, $7a$ with $a = 0.7$).

b) $0.7 + 2.1 + 3.5 + 4.9 = 11.2$. Check: $0.7(4)^2 = 0.7 \times 16 = 11.2$.

c) All 1.4: each step is 1.4 more than the one before, so the relation is quadratic.

Horizontal translations and vertex form

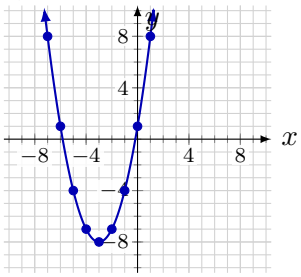
12. The completed table:

	Equation	Vertex	Axis of symmetry	Opens	Max or min value	Stretch, compression, or reflection
a)	$y = 3(x+4)^2 - 1$	$(-4,\ -1)$	$x = -4$	up	min -1	stretch by a factor of 3
b)	$y = -\frac{1}{4}(x-6)^2 + 2$	$(6,\ 2)$	$x = 6$	down	max 2	compression by a factor of $\frac{1}{4}$, and a reflection
c)	$y = -(x-7)^2$	$(7,\ 0)$	$x = 7$	down	max 0	a reflection only
d)	$y = \frac{1}{2}x^2 + 9$	$(0,\ 9)$	$x = 0$	up	min 9	compression by a factor of $\frac{1}{2}$

13. The vertex and the step points are marked.

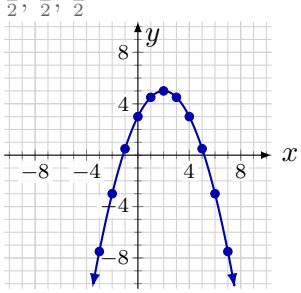
a) $y = (x+3)^2 - 8$

vertex $(-3, -8)$; steps up 1, 3, 5, 7



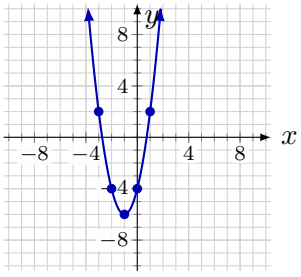
b) $y = -\frac{1}{2}(x-2)^2 + 5$

vertex $(2, 5)$; steps down $\frac{1}{2}$, $\frac{3}{2}$, $\frac{5}{2}$, $\frac{7}{2}$, $\frac{9}{2}$



c) $y = 2(x+1)^2 - 6$

vertex $(-1, -6)$; steps up 2, 6; the next, up 10, leaves the grid



14. a) $y = -\frac{1}{3}(x+6)^2 + 4$

b) $y = (x-5)^2 - 2$

c) vertical stretch by a factor of 4, 2 units left, and 7 units down

d) reflection in the x-axis, vertical stretch by a factor of 3, 1 unit right, and 8 units up

15. The bracket $x-4$ is zero at $x = 4$, so $h = 4$ and the vertex is $(4,\ 3)$: 4 units right, not left. $a = -1 < 0$, so the parabola opens down and 3 is its **maximum** value.

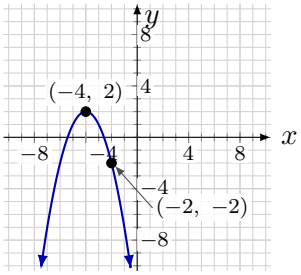
Mixed review

16.

x	y	First differences	Second differences
0	0.0		
		1.7	
1	1.7		2.9
		4.6	
2	6.3		3.6
		8.2	
3	14.5		2.8
		11.0	
4	25.5		3.6
		14.6	
5	40.1		

Close to quadratic. The first differences grow, so it is not linear. The second differences (2.9, 3.6, 2.8, 3.6) are only roughly constant, about 3.2, because the distances are measured data.

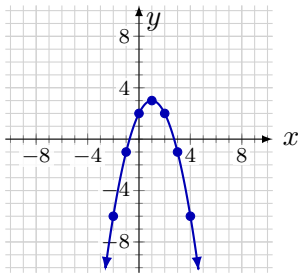
17.



$y = -(x+4)^2 + 2$: it opens down, so $a = -1$; the vertex is $(-4,\ 2)$.
Check: $-(-2+4)^2 + 2 = -4 + 2 = -2$.

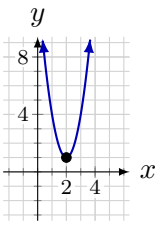
18. $y = -(x-1)^2 + 3$. The second differences are all -2, so it is quadratic.

x	y	First differences	Second differences
−2	−6		
		5	
−1	−1		−2
		3	
0	2		−2
		1	
1	3		−2
		−1	
2	2		−2
		−3	
3	−1		−2
		−5	
4	−6		

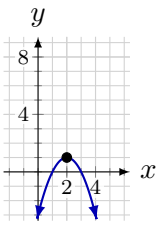


19. A is (ii), B is (iii), C is (iv), and D is (i). From the vertex, 1 across: A rises 3, C rises $\frac{1}{3}$ (it is the widest), D rises 1, and B goes down 1. Equation (ii) has $a = 3$, so its graph is the narrowest, A. Equation (iv) has $a = \frac{1}{3}$, so its graph is the widest, C.

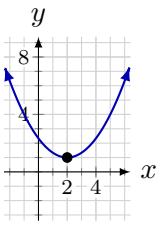
A



B



C



D

