Summary – Analysing Vertex Form and Equations from Graphs

Adapted from Principles of Mathematics 10, McGraw-Hill Ryerson, 2007.

The vertex and one more point are enough to write the equation of a parabola. The axis of symmetry gives more points with no arithmetic.

Two ways to find a

The vertex $(h,\ k)$ gives $y = a(x-h)^2 + k$ with only a unknown.

By the step pattern. From the vertex, 1 across goes up $1a$, and the next 1 across goes up $3a$ (down when $a < 0$). For example, a parabola with vertex $(2,\ -1)$ that passes through $(3,\ 1)$ and $(4,\ 7)$ goes up 2, then 6, so $a = 2$ and $y = 2(x-2)^2 - 1$.

By substitution. Put the other point into the equation and solve for a. This always works, even when the point is far from the vertex.

When steps fail. The step pattern needs y-values you can read exactly one step and two steps from the vertex. When they fall between grid lines, as they do when $a = -\frac{2}{3}$, substitute a point instead.

An Equation from a Vertex and a Point

A parabola has vertex $(-2,\ 1)$ and passes through $(1,\ -5)$.

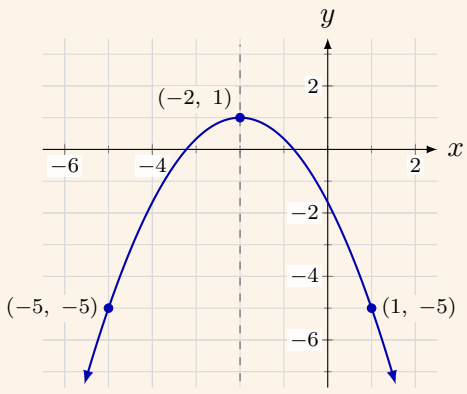
The vertex gives $y = a(x+2)^2 + 1$. Substitute $(1,\ -5)$:

$$\begin{aligned} -5 &= a(1+2)^2 + 1 \\ -5 &= 9a + 1 \\ -6 &= 9a, \quad \text{so } a = -\tfrac{2}{3}. \end{aligned}$$

$$y = -\tfrac{2}{3}(x+2)^2 + 1$$

Check with the partner of $(1,\ -5)$ across the axis $x = -2$, the point $(-5,\ -5)$: $-\frac{2}{3}(-5+2)^2 + 1 = -6 + 1 = -5$.

One step from the vertex the parabola goes down $\frac{2}{3}$, between grid lines, so the step pattern cannot be read here.



Why "vertical stretch"?

$y = 4x^2$ is 4 times as high as $y = x^2$ at every x. It is also squeezed toward the y-axis, to half the width. Both descriptions fit. We use the vertical one because a multiplies every y-value, so the factor shows in the equation.

The y-intercept, its partner, and the axis of symmetry

The y-intercept does not show in vertex form. Substitute $x = 0$. For $y = 2(x-2)^2 - 1$: $y = 2(0-2)^2 - 1 = 7$, the point $(0,\ 7)$.

The axis of symmetry is $x = 2$. The point $(0,\ 7)$ is 2 units left of it, so the point 2 units right, $(4,\ 7)$, is at the same height. No arithmetic is needed.

Why the name? The vertical line through the vertex cuts the parabola into two mirror halves. Fold the graph along it and each point lands on its partner: $(1,\ 1)$ on $(3,\ 1)$, $(0,\ 7)$ on $(4,\ 7)$. Two x-values the same distance from h make $(x-h)^2$ the same number, so they give the same y.

