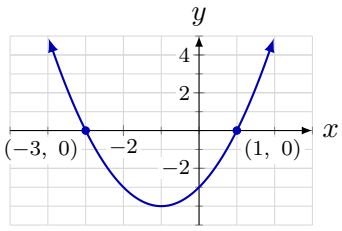
Summary – Intercepts Form

Adapted from Principles of Mathematics 10, McGraw-Hill Ryerson, 2007.

Vertex form shows where the vertex is. A second form, with two brackets, shows where the parabola crosses the x-axis.

Where $y = 0$: the zeros



1. The graph (Card 1). Every point on the x-axis has $y = 0$, so $y = 0$ where the graph crosses the x-axis: at $(-3,\ 0)$ and $(1,\ 0)$.

x	$x-1$	$x+3$	y
-4	-5	-1	5
-3	-4	**0**	**0**
-2	-3	1	-3
-1	-2	2	-4
0	-1	3	-3
1	**0**	4	**0**
2	1	5	5

2. The table (Card 2). The rows where $y = 0$ have the same x-values, -3 and 1. In each of those rows one bracket is 0, and 0 times any number is 0, so y is 0.

3. The algebra (Card 3). On the x-axis, $y = 0$, so put $y = 0$ into the equation:

$$0 = (x-1)(x+3)$$
$$x-1=0 \quad \text{or} \quad x+3=0$$
$$x=1 \quad \text{or} \quad x=-3$$

A product is 0 only when one of the numbers being multiplied is 0, because two numbers that are not 0 never multiply to make 0. So one bracket must be 0.

The **zeros** of a relation are the x-values where $y = 0$. Each zero makes one bracket equal to 0. In $x + 3$, the number 3 has a plus sign, so the zero is -3.

The a value never moves the zeros (Card 4). In $y = 2(x-4)(x+5)$, 2 is never 0, so $x - 4 = 0$ or $x + 5 = 0$: the zeros are 4 and -5, the same as for $y = (x-4)(x+5)$. The a value multiplies every y-value (a vertical stretch), and $2 \times 0 = 0$, so the points on the x-axis stay where they are.

Intercepts Form: $y = a(x - r)(x - s)$

Zeros: r and s. The graph crosses the x-axis at $(r,\ 0)$ and $(s,\ 0)$.

Axis of symmetry: a parabola is symmetric, and the two points where the graph crosses the x-axis are both at height 0, so the axis is **midway** between them: $x = \dfrac{r + s}{2}$.

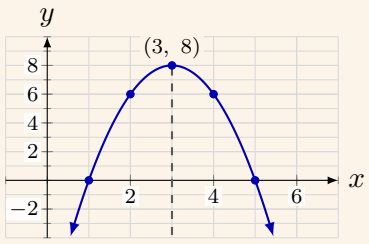
Vertex: on the axis. Substitute the axis's x-value into the equation to find the vertex's y-value.

The a value: as in vertex form. $a > 0$ opens upward and $a < 0$ opens downward. a stretches or compresses the graph vertically, but it never moves the zeros. To find a, substitute one more point that is *not* on the x-axis.

To sketch: plot the points where the graph crosses the x-axis, find the axis midway, substitute to find the vertex, read the direction from the sign of a, and check one more point and its partner across the axis.

Worked example. A parabola has zeros 1 and 5. It passes through $(2,\ 6)$. Find its equation and its vertex.

$y = a(x - 1)(x - 5)$
$6 = a(2 - 1)(2 - 5) = -3a$, so $a = -2$.
$y = -2(x - 1)(x - 5)$
Axis: $x = \frac{1+5}{2} = 3$. Vertex:
$-2(2)(-2) = 8$, so $(3,\ 8)$, a maximum.
Check the partner $(4,\ 6)$: $-2(3)(-1) = 6$.



Which form suits the information?

You know	Start from	Then
the vertex and one more point	$y = a(x - h)^2 + k$	substitute the point to find a
the zeros and one more point	$y = a(x - r)(x - s)$	substitute the point to find a
the zeros and the maximum or minimum	$y = a(x - r)(x - s)$	the vertex is midway; substitute it