Summary – Modelling with Vertex Form and Intercepts Form

Adapted from Principles of Mathematics 10, McGraw-Hill Ryerson, 2007.

A quadratic model starts from what the situation tells you. The information you have points to the form.

Which form the information gives you

Vertex form, $y = a(x - h)^2 + k$

Use it when you know the highest or lowest point: a greatest height and when or where it happens.

Intercepts form, $y = a(x - r)(x - s)$

Use it when you know where the path or the arch meets the ground: the two ends of an arch, where a path starts and lands.

Either way, one more point gives a. Always say what x and y stand for, with units.

A Coin in Vertex Form

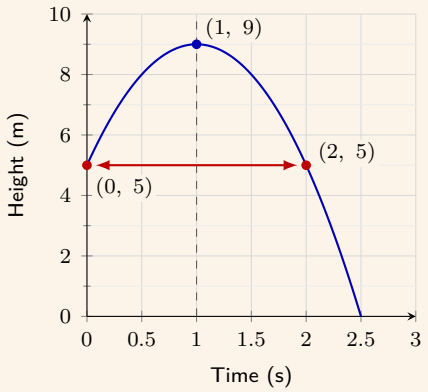
A coin is tossed up from a bridge, 5 m above a river. It reaches its greatest height, 9 m, 1 s after it is tossed.

The greatest height is the vertex, $(1,\ 9)$. The toss gives the point $(0,\ 5)$: $5 = a(0 - 1)^2 + 9$, so $a = -4$.

$$y = -4(x - 1)^2 + 9$$

x is the time in seconds; y is the height above the river in metres. $h = 1$: the coin is highest after 1 s. $k = 9$: its greatest height is 9 m. a is negative: the coin rises, then falls.

The matching time. The axis of symmetry is $x = 1$. The coin is 5 m high at 0 s, 1 s before the peak, so it is 5 m high again 1 s after the peak, at 2 s. No substitution is needed.



A Dolphin's Jump in Intercepts Form

A dolphin leaves the water and goes back in 4 m away. At its highest it is 2 m above the water. Put the point where it leaves the water at the origin. The zeros are 0 and 4, so $y = ax(x - 4)$. The top is midway, at $(2,\ 2)$: $2 = a(2)(2 - 4) = -4a$, so $a = -0.5$, and $y = -0.5x(x - 4)$.

The zeros, 0 and 4, are the x-values of the points where the dolphin leaves and enters the water, 4 m apart. a is negative, so the path has a top. At the centre, $x = 2$, the part $x(x - 4)$ gives $(2)(2 - 4) = -4$. The a value multiplies that: $-0.5 \times (-4) = 2$, the greatest height of the jump. So the size of a sets how high the jump is. With the same zeros, doubling the a value would double the height.

How far to trust a model

A model describes its situation only inside the values that make sense for it. A prediction inside those values is an **interpolation**, and it can be trusted: the coin at 0.5 s is $-4(0.5 - 1)^2 + 9 = 8$ m high. A prediction outside them is an **extrapolation**, and it may make no sense: at 3 s the model gives $-4(3-1)^2+9 = -7$ m, below the river. The coin reaches the water at 2.5 s, so the model holds only from 0 s to 2.5 s.