Summary – Negative and Zero Exponents

Adapted from Principles of Mathematics 10, McGraw-Hill Ryerson, 2007.

Two routes lead to the same meaning for a zero or negative exponent: carry on a pattern, or divide powers by the exponent law.

The pattern route

Power	Value
2^3	8
2^2	4
2^1	2
2^0	1
2^{-1}	$\frac{1}{2}$
2^{-2}	$\frac{1}{4}$
2^{-3}	$\frac{1}{8}$

Each step down the table lowers the exponent by 1 and **divides the value by 2**. Nothing stops the pattern at 2^1: one more step gives $2 \div 2 = 1$, so $2^0 = 1$. Then come $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, and so on.

Set $2^3 = 8$ beside $2^{-3} = \frac{1}{8}$: the negative exponent gives **one over** the matching positive power, $2^{-3} = \dfrac{1}{2^3}$.

Powers of 3 behave the same way, dividing by 3 each step: 27, 9, 3, 1, $\frac{1}{3}$, $\frac{1}{9}$, $\frac{1}{27}$. So $3^0 = 1$ and $3^{-2} = \dfrac{1}{3^2} = \dfrac{1}{9}$.

The law route

The exponent law for division, $a^m \div a^n = a^{m-n}$, must agree with ordinary arithmetic.

Zero: $2^3 \div 2^3 = 2^{3-3} = 2^0$,
and also $8 \div 8 = 1$.

So $2^0 = 1$. Any number except 0 divided by itself is 1, so the same holds for every base but 0.

Negative: $2^2 \div 2^5 = 2^{2-5} = 2^{-3}$, and also

$$\frac{2 \times 2}{2 \times 2 \times 2 \times 2 \times 2} = \frac{1}{2 \times 2 \times 2} = \frac{1}{2^3}.$$

So $2^{-3} = \dfrac{1}{2^3} = \dfrac{1}{8}$: the same answer as the pattern.

Zero and Negative Exponents

For any base a that is not 0, and any whole number n:

$$a^0 = 1 \qquad a^{-n} = \frac{1}{a^n}$$

Worked example. Find the value of $3^{-2} + 3^0$.

$3^{-2} = \dfrac{1}{3^2} = \dfrac{1}{9}$, and $3^0 = 1$.

$3^{-2} + 3^0 = \dfrac{1}{9} + 1 = \dfrac{1}{9} + \dfrac{9}{9} = \dfrac{10}{9}$.

Brackets and fractions. $(-3)^{-2} = \dfrac{1}{(-3)^2} = \dfrac{1}{9}$. A fraction flips: $\left(\frac{1}{2}\right)^{-1} = 2$.

Two common errors.

1. "$2^{-3} = -8$." No: the negative sign in the exponent means "one over", not "make the answer negative". $2^{-3} = \frac{1}{8}$, a positive number. Halving a positive number never gives a negative one.

2. "$5^0 = 0$." No: going down the powers of 5, $5 \div 5 = 1$; and $5^2 \div 5^2 = 5^0 = 25 \div 25 = 1$. A product with no factors of 5 is left at its start, 1, not 0.