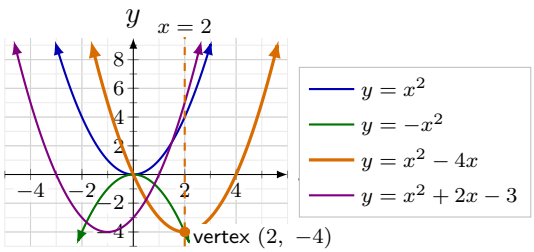
Summary – Quadratic Relations and Finite Differences

Adapted from Principles of Mathematics 10, McGraw-Hill Ryerson, 2007.

Each group tabled four relations and graphed each on its own grid. The graphs shared one shape; the tables shared one pattern.

One shape: the parabola



Each graph is a U-shaped curve, a **parabola**.
For $y = x^2 - 4x$ (orange):

- It **opens up**. Only $y = -x^2$ **opens down**.
- Its turning point, the **vertex**, is $(2,\ -4)$.
- It is symmetric about the vertical line through the vertex, the **axis of symmetry**, $x = 2$.
- It opens up, so -4 is its **minimum value**. The vertex of $y = -x^2$ gives a **maximum value**, 0.
- It crosses the y-axis once, at its y**-intercept**, 0.

The y-intercept from the equation

The y-intercept is the value of y when $x = 0$. Every term with an x becomes 0, leaving the constant term: for $y = x^2 + 2x - 3$, $y = 0 + 0 - 3 = -3$. The other three have no constant term, so they cross at 0. The four y-intercepts are 0, 0, 0, and -3.

One pattern: finite differences

With evenly spaced x values, the **first differences** are differences of consecutive y values; the **second differences** are differences of consecutive first differences.

Linear: $y = 3x - 2$

x	y	First differences	Second differences
0	−2		
		$1 - (-2) = 3$	
1	1		$3 - 3 = 0$
		$4 - 1 = 3$	
2	4		$3 - 3 = 0$
		$7 - 4 = 3$	
3	7		$3 - 3 = 0$
		$10 - 7 = 3$	
4	10		

Each difference is the lower value minus the one above it.

Quadratic: $y = x^2 - 4x$

x	y	First differences	Second differences
0	0		
		$-3 - 0 = -3$	
1	−3		$-1 - (-3) = 2$
		$-4 - (-3) = -1$	
2	−4		$1 - (-1) = 2$
		$-3 - (-4) = 1$	
3	−3		$3 - 1 = 2$
		$0 - (-3) = 3$	
4	0		

Each difference is the lower value minus the one above it.

The Rule

Linear: the first differences are constant. **Quadratic:** the first differences are not constant, but the second differences are. **Neither:** neither column is constant.

The rule needs **evenly spaced** x values; with uneven steps a quadratic can look like "neither". Measured data, like the palm data, give second differences that are only *approximately* constant.

Four Ways to Describe a Relation: $y = x^2 - 4x$

Any relation can be described in words, as a table of values, as an equation, and as a graph.

In words

A parabola that opens up, with its vertex at $(2,\ -4)$, its axis of symmetry $x = 2$, and its y-intercept 0.

As an equation

$y = x^2 - 4x$

As a table of values

x	y
−1	5
0	0
1	−3
2	−4
3	−3
4	0
5	5

As a graph

