Summary – Solving Linear Systems Graphically

Solving and analyzing linear systems of equations: what the terms mean, how to solve a system by graphing, and how to tell how many solutions a system has without graphing.

Solving linear systems

What is a **linear system**? It is simply two equations of lines. For example:

$$y = 4x + 3 \qquad y = -2x + 1$$

What does it mean to **solve** a linear system of equations? It just means to find where the graphs of the equations cross. That is, to find the coordinates of the point where the two lines meet.

Example 1

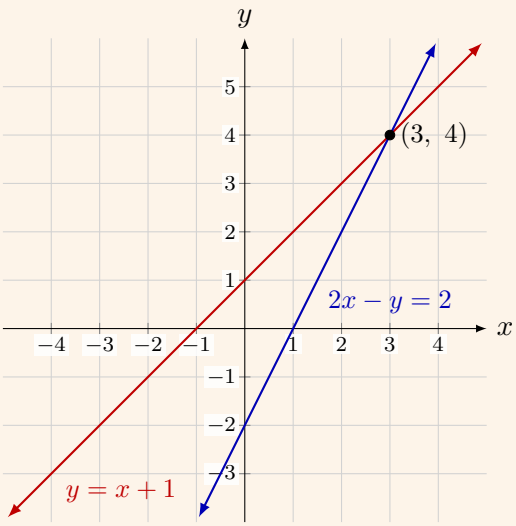
Solve the given linear system by graphing.

$$y = x + 1$$
$$2x - y = 2$$

First, we graph each equation.
The first equation is in $y = mx + b$ form. So, graph by plotting the y-intercept, b. Then use the slope, m, and $\frac{\text{rise}}{\text{run}}$ to plot a few more points. Then draw the line.

The second equation has terms with both x and y on the left side. So, graph using the x- and y-intercepts. Use the "cover up" shortcut we discussed last week: complete the mini-table, plot the intercepts, and draw the line.

x	y
0	-2
1	**0**



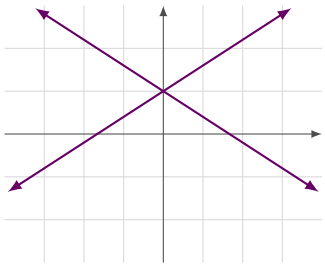
Now, look at the graph and find where the lines meet.

The lines meet at $(3,\ 4)$.

$\therefore$ the solution to the linear system is $(3,\ 4)$.

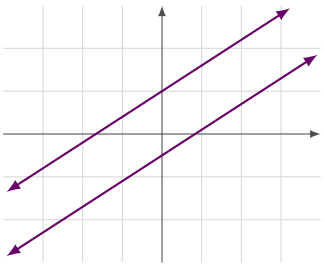
Analyzing linear systems

There are three ways that two lines can meet, as shown below.
You can identify each case by analyzing the slope and y-intercept of each line.



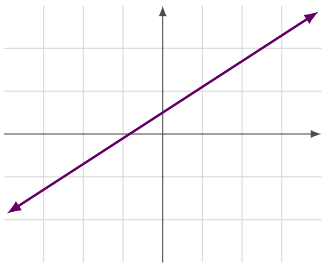
intersection
slope: different
y-int: different

one sol'n
(lines cross at one point)



parallel
slope: same
y-int: different

no sol'ns
(lines do not cross)



parallel, coincident
slope: same
y-int: same

infinite sol'ns
(lines cross at infinitely many points)

Example 2

Without graphing, state whether each system of equations has one solution, no solutions, or infinitely many solutions.

a) $y = -2x + 3$
$y = -2x + 1$

slope same
y-int different
$\therefore$ no sol'ns

b) $-x + 2y = -6$
$2x + y = -1$

Solve each equation for y so that you can analyze it.

$-x + 2y = -6$
$2y = x - 6$
$y = \frac{1}{2}x - \frac{6}{2}$
$y = \frac{1}{2}x - 3$

$2x + y = -1$
$y = -2x - 1$

slope different
(now, stop! as soon as you see the slope is different, you know there is just one sol'n)
$\therefore$ one sol'n