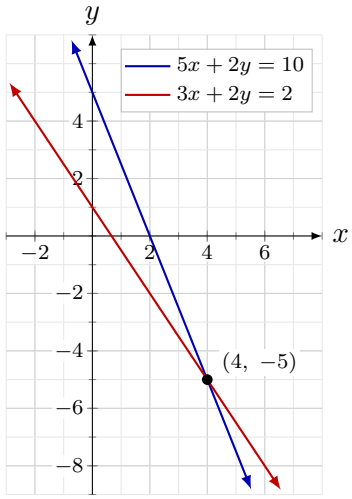
Summary – Solving Linear Systems by Elimination

Why graphing and substitution can be awkward for some systems, and how to solve a linear system by adding or subtracting its equations.

When other methods are awkward

Consider the following linear system:

$$\begin{aligned} 5x + 2y &= 10 \quad \textbf{A} \\ 3x + 2y &= 2 \quad \textbf{B} \end{aligned}$$



Solving this linear system by graphing, the solution appears to be $(4,\ -5)$.

Now consider solving the linear system by substitution. Does it look like it will be easy to solve using this method?

Not really. No matter which variable we isolate, we end up with a **fraction for a coefficient**. For example, from **A**:

$$\begin{aligned} 5x + 2y &= 10 \\ 2y &= -5x + 10 \\ y &= -\tfrac{5}{2}x + 5 \end{aligned}$$

There is another method. It is called **solving linear systems by elimination**. Here is how it works:

1. Look for terms that have **matching coefficients**.
2. Add or subtract the equations to **eliminate** a variable.
3. Continue to solve for the variable that is left.
4. Substitute the value you found into either equation, and solve for the other variable.

Example 1

Solve the linear system above by elimination.

The y-terms have matching coefficients, so subtract **B** from **A**.

$$\begin{array}{rl} 5x + 2y = 10 & \textbf{A} \\ -\ 3x + 2y = 2 & \textbf{B} \\ \hline 2x = 8 & \\ \frac{2x}{2} = \frac{8}{2} & \\ x = 4 & \end{array}$$

Substitute $x = 4$ into **B**.

$$\begin{aligned} 3x + 2y &= 2 \\ 3(4) + 2y &= 2 \\ 12 + 2y - 12 &= 2 - 12 \\ 2y &= -10 \\ \frac{2y}{2} &= \frac{-10}{2} \\ y &= -5 \end{aligned}$$

$\therefore$ the solution is $(4,\ -5)$, as the graph suggested.

Example 2

Solve by elimination. The x-terms have opposite coefficients, so add the equations.

$$\begin{array}{rrrl} & 4x + 5y = & 7 & \textbf{A} \\ + & -4x - 2y = & -1 & \textbf{B} \\ \hline & 3y = & 6 & \\ & \frac{3y}{3} = & \frac{6}{3} & \\ & y = & 2 & \end{array}$$

Substitute $y = 2$ into **A**.

$$\begin{aligned} 4x + 5y &= 7 \\ 4x + 5(2) &= 7 \\ 4x + 10 - 10 &= 7 - 10 \\ 4x &= -3 \\ \frac{4x}{4} &= \frac{-3}{4} \\ x &= -\frac{3}{4} \end{aligned}$$

$\therefore$ the solution is $\left(-\frac{3}{4},\ 2\right)$. Fractions are OK!

Example 3

Solve by elimination.

$$\begin{aligned} 2x - 3y &= 2 \quad \textbf{A} \\ 5x + 6y &= 5 \quad \textbf{B} \end{aligned}$$

No coefficients match. But what if we multiply both sides of **A** by 2?

As long as we multiply both sides by the same value, the equation stays equivalent.

Now we can add the equations to eliminate y.

$$\begin{array}{rrrl} & 4x - 6y = & 4 & \textbf{A} \times 2 \\ + & 5x + 6y = & 5 & \textbf{B} \\ \hline & 9x = & 9 & \\ & \frac{9x}{9} = & \frac{9}{9} & \\ & x = & 1 & \end{array}$$

Substitute $x = 1$ into **B**.

$$\begin{aligned} 5x + 6y &= 5 \\ 5(1) + 6y &= 5 \\ 5 + 6y - 5 &= 5 - 5 \\ 6y &= 0 \\ \frac{6y}{6} &= \frac{0}{6} \\ y &= 0 \end{aligned}$$

$\therefore$ the solution is $(1,\ 0)$.

Example 4

Solve by elimination.

$$\begin{aligned} 3x - 7y &= 13 \quad \textbf{A} \\ 4x - 5y &= 13 \quad \textbf{B} \end{aligned}$$

If needed to get matching coefficients, we can multiply **both** equations.

Now we can subtract to eliminate x.

$$\begin{array}{rrrl} & 12x - & 28y = & 52 & \textbf{A} \times 4 \\ - & 12x - & 15y = & 39 & \textbf{B} \times 3 \\ \hline & & -13y = & 13 & \\ & & \frac{-13y}{-13} = & \frac{13}{-13} & \\ & & y = & -1 & \end{array}$$

Substitute $y = -1$ into **A**.

$$\begin{aligned} 3x - 7y &= 13 \\ 3x - 7(-1) &= 13 \\ 3x + 7 - 7 &= 13 - 7 \\ 3x &= 6 \\ \frac{3x}{3} &= \frac{6}{3} \\ x &= 2 \end{aligned}$$

$\therefore$ the solution is $(2,\ -1)$.