Summary – Stretches, Compressions, Reflections, and the Step Pattern

Adapted from Principles of Mathematics 10, McGraw-Hill Ryerson, 2007.

Last class, k slid $y = x^2$ up or down. Today's number, a, multiplies every y-value: it changes the shape.

The effect of a in $y = ax^2$

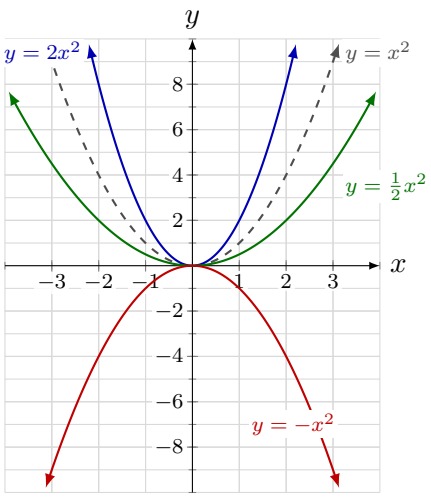
In $y = ax^2$, every y-value of $y = x^2$ is multiplied by a, so every point moves straight up or down, away from the x-axis or toward it.

Vertical stretch ($a > 1$ or $a < -1$): points move away from the x-axis, so the graph is narrower. $y = 2x^2$ doubles every y-value: a vertical stretch by a factor of 2.

Vertical compression ($-1 < a < 1$, $a \neq 0$): points move toward the x-axis, so the graph is wider. $y = \frac{1}{2}x^2$ halves every y-value: a vertical compression by a factor of $\frac{1}{2}$.

Reflection in the x-axis ($a < 0$): every y-value changes sign, so the graph flips over the x-axis and opens **down**; the vertex becomes a **maximum**. $y = -x^2$ is $y = x^2$ reflected; $y = -2x^2$ is stretched by 2 *and* reflected.

The **size** of a sets the width; the **sign** of a sets the direction. $y = 2x^2$ and $y = -2x^2$ are equally wide. What a leaves alone: the vertex $(0,\ 0)$ and the axis of symmetry $x = 0$. Watch the order of operations: $-x^2$ means $-(x^2)$, so at $x = -3$ it is -9.



The Step Pattern: Sketching $y = ax^2$ Without a Table

Starting at the vertex and stepping right one unit at a time, the first differences of $y = ax^2$ are $a,\ 3a,\ 5a,\ 7a, \ldots$ (for $y = x^2$: 1, 3, 5, 7). So to sketch, plot the vertex, then go **over 1 and up a, over 1 and up $3a$, over 1 and up $5a$**, and mirror the points on the other side of the axis of symmetry. When $a < 0$, each step goes down.

$y = x^2$

x	y	step
0	0	
1	1	up 1
2	4	up 3
3	9	up 5

$y = 2x^2$

x	y	step
0	0	
1	2	up 2
2	8	up 6
3	18	up 10

$y = \frac{1}{2}x^2$

x	y	step
0	0.0	
1	0.5	up 0.5
2	2.0	up 1.5
3	4.5	up 2.5

$y = -x^2$

x	y	step
0	0	
1	−1	down 1
2	−4	down 3
3	−9	down 5

With a translation, $y = ax^2 + k$: start at the vertex $(0,\ k)$ and use the same steps. For $y = -2x^2 + 7$: vertex $(0,\ 7)$, then down 2 to $(\pm 1,\ 5)$, then down 6 to $(\pm 2,\ -1)$. On a -10 to 10 grid, stop when the next step would leave the grid.

Why it works: going from $x = n - 1$ to $x = n$, y changes by $an^2 - a(n-1)^2 = a(2n-1)$, which is $a,\ 3a,\ 5a,\ \ldots$ for $n = 1, 2, 3, \ldots$

Finding a from a graph

For a parabola with its vertex at the origin through $(1,\ 2)$ and $(2,\ 8)$: **by the step pattern**, the first step from the vertex rises 2, so $a = 2$ (and the next rises 6, which is $3a$); **by substitution**, $2 = a(1)^2$ gives

$a = 2$. Either way $y = 2x^2$; check another point: $2(2)^2 = 8$.

Four Ways to Describe a Relation

A relation can be given in words, as a table of values, as an equation, and as a graph. All four describe the same relation. Here is $y = 2x^2 - 3$ all four ways.

In words

The parent parabola stretched vertically by a factor of 2, then translated 3 units down. It opens upward, its vertex is $(0,\ -3)$, its minimum value is -3, its axis of symmetry is $x = 0$, and it crosses the x-axis at about $x = -1.2$ and $x = 1.2$.

Table of values

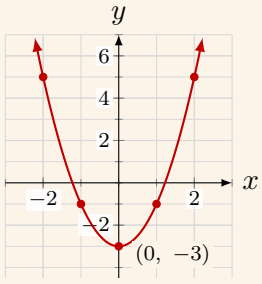
x	y
-2	5
-1	-1
0	-3
1	-1
2	5

Equation

$$y = 2x^2 - 3$$

Square x, double it, then subtract 3.

Graph



Each form shows something easily. The words say how the graph is made from $y = x^2$. The table gives exact points; from the vertex its steps are up 2, then up 6 (a, then $3a$, with $a = 2$). The equation shows both numbers at once: the 2 is the stretch, and the -3 is the vertex's y-coordinate. The graph shows the shape at a glance, narrower than $y = x^2$ and 3 units lower.