Summary – The Parent Parabola and Vertical Translations

Adapted from Principles of Mathematics 10, McGraw-Hill Ryerson, 2007.

Every parabola in this thread is built from one: $y = x^2$. Adding a number k slides it up or down and changes nothing else.

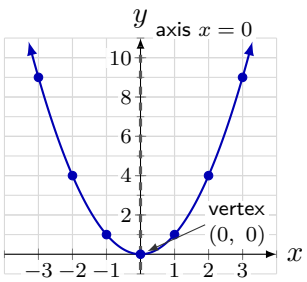
The parent parabola, $y = x^2$

x	y
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9

Squaring a number and its opposite gives the same result, so the table of values is symmetric.

The graph is a **parabola** that opens upward. Its lowest point, the **vertex**, is $(0,\ 0)$, so its **minimum value** is 0. It is symmetric about the **axis of symmetry**, the vertical line $x = 0$: fold the graph along the y-axis and the two halves match.

The first differences are $-5,\ -3,\ -1,\ 1,\ 3,\ 5$, and the **second differences** are all 2. Constant second differences mark a **quadratic relation**.



Vertical Translations: $y = x^2 + k$

Adding k to every y-value of $y = x^2$ slides the whole graph k units: **up** when $k > 0$, **down** when $k < 0$. This is a **vertical translation**.

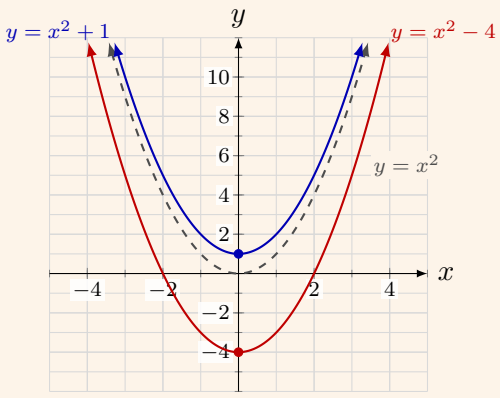
$y = x^2 + 1$ is $y = x^2$ moved 1 up, vertex $(0,\ 1)$; $y = x^2 - 4$ is $y = x^2$ moved 4 down, vertex $(0,\ -4)$.

What k changes: the vertex is $(0,\ k)$; the y-intercept is k; the minimum value is k.

What k leaves alone: the shape, the direction of opening, the axis of symmetry $x = 0$, and the first and second differences (still 2). Only the y-values move.

So we can sketch $y = x^2 + k$ without a table: plot the vertex $(0,\ k)$, then step out 1 across and 1 up, 2 across and 4 up, 3 across and 9 up, on each side.

When $k < 0$ the parabola crosses the x-axis twice ($y = x^2 - 9$ at $x = \pm 3$, since $3^2 - 9 = 0$); when $k = 0$ it touches it at the origin; when $k > 0$ it never meets it.



Back to the rolling can

The can's distance down the ramp, plotted against time, made half a parabola with its vertex at the origin, the same position as the vertex of $y = x^2$: distance is a quadratic relation of time. Time plotted against distance shows the same relation with the axes swapped, so the curve looks different. But with distance in centimetres and time in seconds, the can's curve was much steeper than $y = x^2$ (a roll of 100 cm took about 1.8 s, and 1.8^2 is only about 3). Adding k only slides a graph up or down, so it cannot change that steepness.

Four Ways to Describe a Relation

A relation can be given in words, as a table of values, as an equation, and as a graph. All four describe the same relation. Here is $y = x^2 - 4$ all four ways.

In words

The parent parabola slid down 4 units. It opens upward, its vertex is $(0,\ -4)$, its axis of symmetry is $x = 0$, and it crosses the x-axis at $x = -2$ and $x = 2$.

Table of values

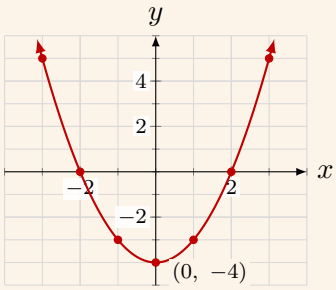
x	y
-3	5
-2	0
-1	-3
0	-4
1	-3
2	0
3	5

Equation

$$y = x^2 - 4$$

Square x, then subtract 4.

Graph



Each form makes something easy to see. The words say how the graph was made from $y = x^2$. The table gives exact points, and its second differences (all 2) show it is quadratic. The equation gives the y-value for any x, and its -4 is the vertex's y-coordinate. The graph shows the whole shape at a glance, and where it crosses the axes. Moving from one form to another is a good way to check your work.