Summary – Transformations of the Quadratic Relation

Adapted from Principles of Mathematics 10, McGraw-Hill Ryerson, 2007.

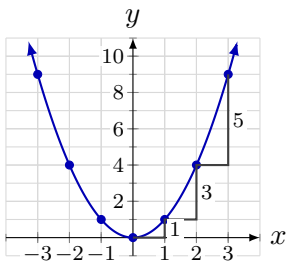
Every parabola this week is the parent parabola $y = x^2$, moved or reshaped. Three numbers, a, h, and k, say exactly how.

The parent parabola, $y = x^2$

x	y
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9

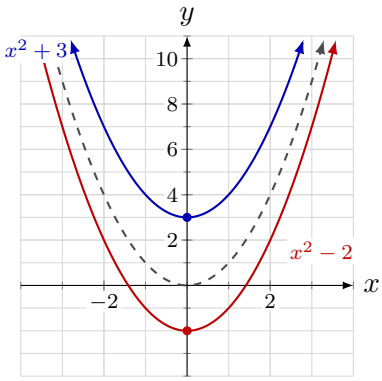
The vertex is $(0,\ 0)$, the axis of symmetry is $x = 0$, and the parabola opens upward, with a minimum value of 0.

The step pattern. From the vertex, step 1 across and 1 up, then 1 more across and 3 more up, then 1 across and 5 up: the first differences 1, 3, 5. Mirror the points in the axis of symmetry to get the other half. For $y = ax^2$ every step is multiplied by a: $1a$, $3a$, $5a$, ...



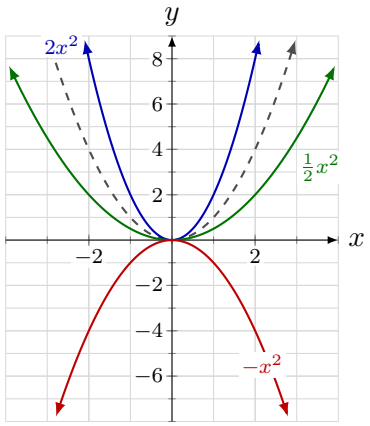
What each number does

k: up and down



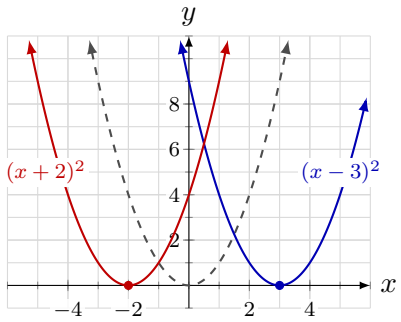
$y = x^2 + k$ moves $y = x^2$ **up** k units when $k > 0$ and **down** $|k|$ units when $k < 0$: a vertical translation. The vertex moves to $(0,\ k)$.

a: shape and direction



$y = ax^2$ is a vertical **stretch** when $a > 1$ or $a < -1$ (narrower), a vertical **compression** when a is between -1 and 1, not 0 (wider), and a **reflection** in the x-axis when $a < 0$ (it opens down).

h: left and right

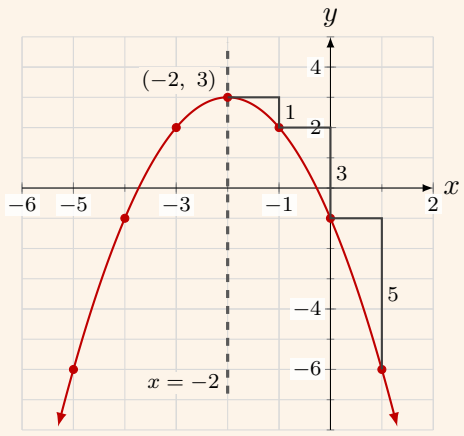


$y = (x - h)^2$ moves $y = x^2$ **right** h units when $h > 0$ and **left** when $h < 0$: a **horizontal translation**. The vertex is where the bracket is zero, so $(x - 3)^2$ moves right 3 and $(x + 2)^2$ moves left 2.

Vertex Form: $y = a(x - h)^2 + k$

Put the three together. Starting from $y = x^2$: stretch or compress by the size of a (reflect if $a < 0$), then translate h units across and k units up or down.

Vertex	$(h,\ k)$
Axis of symmetry	$x = h$
Opens	up when $a > 0$; down when $a < 0$
Max or min	minimum k when $a > 0$; maximum k when $a < 0$



Watch the sign of h. $y = (x + 2)^2 + 3$ is $y = (x - (-2))^2 + 3$, so $h = -2$ and the vertex is $(-2,\ 3)$.

Sketching by the step pattern: $y = -(x + 2)^2 + 3$.

1. Plot the vertex $(-2,\ 3)$.
2. $a = -1$, so the steps are -1, -3, -5: from the vertex, 1 right and 1 down to $(-1,\ 2)$, 1 right and 3 down to $(0,\ -1)$, 1 right and 5 down to $(1,\ -6)$.
3. Mirror these in the axis $x = -2$: $(-3,\ 2)$, $(-4,\ -1)$, $(-5,\ -6)$.
4. Join the points with a smooth curve. Check one: at $x = 0$, $-(0 + 2)^2 + 3 = -1$.

Four Ways to Describe a Relation

A relation can be given in words, as a table of values, as an equation, and as a graph. All four describe the same relation. Here is $y = 2(x - 1)^2 - 3$ all four ways.

In words

The parent parabola stretched vertically by a factor of 2, then moved 1 unit right and 3 units down. It opens upward, its vertex is $(1,\ -3)$, its axis of symmetry is $x = 1$, and its minimum value is -3.

Table of values

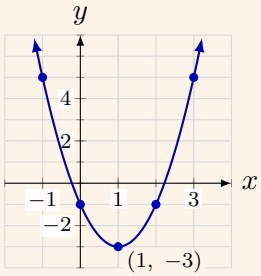
x	y
-1	5
0	-1
1	-3
2	-1
3	5

Equation

$$y = 2(x - 1)^2 - 3$$

Subtract 1, square, double, then subtract 3.

Graph



Each form makes something easy to see. The words say how the graph was made from $y = x^2$. The table gives exact points; read from the vertex row, the y-values rise 2, then 6 more: the step pattern $1a$, $3a$ with $a = 2$. The equation shows the vertex $(1,\ -3)$ and the stretch at a glance. The graph shows the whole shape and where it sits. Moving from one form to another is a good way to check your work.