Horizontal Translations and Vertex Form

Print double-sided (flip on the long edge: Long-Edge binding): three sheets. Cards 1 to 6 are the sequence; Cards 7, 8, and 9 are extensions for groups that finish early. Cut along the frames.

Card 1

$$y = (x - 3)^2$$

x	$x - 3$	y
0	-3	9
1	-2	
2		
3		
4		
5		
6		

On your board, beside your first grid, copy this table of values for $y = (x - 3)^2$ and complete it.

Then plot the points from your table on that grid and join them with a smooth curve.

Where is the vertex? Compared with $y = x^2$, which way did the graph move, and how far?

Does that direction match the sign you see in the equation? Use the $x - 3$ column to explain.

Card 2

On your second grid, make a table of values for

$$y = (x + 2)^2$$

with an $x + 2$ column, for x from -5 to 1. Plot it, and put it beside your graph from Card 1.

Where is the vertex now? Which way did the graph move?

Is that what you expected after Card 1?

Card 3

Cards 1 and 2 both fit the pattern

$$y = (x - h)^2$$

What is h for each?

Write one sentence that says what h does to the graph of $y = x^2$, for any h. Include the vertex and the axis of symmetry.

Someone says $y = (x + 2)^2$ moves 2 to the right "because it says plus". Convince them otherwise.

Horizontal Translations and Vertex Form — Backs

Answers for the cards on the other side of this sheet.

Card 1 — answer

$y = (x - 3)^2$

x	$x - 3$	y
0	−3	9
1	−2	4
2	−1	1
3	0	0
4	1	1
5	2	4
6	3	9

Vertex $(3, 0)$, axis of symmetry $x = 3$. The graph moved **3 units right**, although the equation shows "minus 3".

The $x - 3$ column explains it: y is 0 where $x - 3 = 0$, at $x = 3$. Every y-value of $y = x^2$ still appears, but at an x that is 3 *larger*, so every point is 3 units to the right.

Card 3 — answer

Card 1: $h = 3$. Card 2: $y = (x - (-2))^2$, so $h = -2$.

$y = (x - h)^2$ is $y = x^2$ moved h units sideways: **right** when $h > 0$, **left** when $h < 0$. This is a **horizontal translation**. The vertex is $(h, 0)$; the axis of symmetry is $x = h$.

The vertex sits where the bracket is zero: $x + 2 = 0$ at $x = -2$, which is left of the origin.

Card 2 — answer

x	$x + 2$	y
−5	−3	9
−4	−2	4
−3	−1	1
−2	0	0
−1	1	1
0	2	4
1	3	9

Vertex $(-2, 0)$: the graph moved **2 units left**, the opposite way from Card 1.

y is 0 where $x + 2 = 0$, at $x = -2$. A plus sign inside the brackets moves the graph left.

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Card 4

$$y = (x - 2)^2 + 3$$

Before any table, predict: where is the vertex? What is the axis of symmetry? Where does the graph sit compared with $y = x^2$?

Then, on a new grid, make a table of values for x from 0 to 4 and plot it to test your prediction.

Card 5

$$y = -(x + 1)^2 + 5$$

No table this time. On a new grid, sketch it by the step pattern, starting from the vertex.

Substitute two of your points to check them.

Then describe the graph in words: its vertex, its axis of symmetry, which way it opens, and its maximum or minimum value.

Card 6

Every relation today fits

$$y = a(x - h)^2 + k$$

Say in words what a, h, and k each do to the graph of $y = x^2$.

Give the vertex and the axis of symmetry in terms of h and k.

How can you tell from the equation whether the vertex is a maximum or a minimum?

Horizontal Translations and Vertex Form — Backs

Answers for the cards on the other side of this sheet.

Card 5 — answer

Vertex $(-1, 5)$. Here $a = -1$, so the steps go **down** 1, 3, 5: $(0, 4)$ and $(-2, 4)$, $(1, 1)$ and $(-3, 1)$, $(2, -4)$ and $(-4, -4)$.

Check: $x = 1$: $-(2)^2 + 5 = 1$. $x = -3$: $-(-2)^2 + 5 = 1$.

In words: $y = x^2$ reflected in the x-axis, then moved 1 unit left and 5 units up. Axis of symmetry $x = -1$; it opens down; maximum value 5.

Card 4 — answer

x	$x - 2$	y
0	-2	7
1	-1	4
2	0	3
3	1	4
4	2	7

Vertex $(2, 3)$, axis of symmetry $x = 2$.

In words: $y = x^2$ moved 2 units right and 3 units up. It opens up, keeps the same shape, and has a minimum value of 3.

You now have it in words, as a table of values, as an equation, and as a graph.

Card 6 — answer

$\boldsymbol{a}$: stretches the graph vertically when $a > 1$ or $a < -1$, compresses it when a is between -1 and 1 (not 0), and reflects it in the x-axis when $a < 0$.

$\boldsymbol{h}$: moves it h units right ($h > 0$) or left ($h < 0$).

$\boldsymbol{k}$: moves it k units up ($k > 0$) or down ($k < 0$).

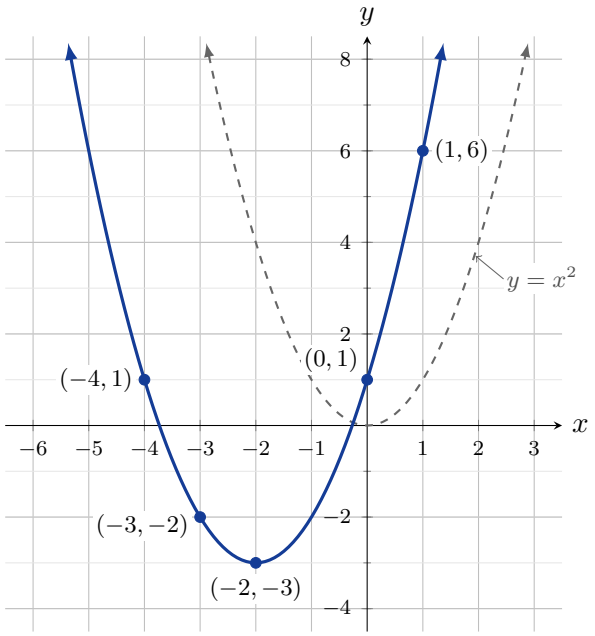
Vertex (h, k); axis of symmetry $x = h$.

$a > 0$: opens up, the vertex is a minimum, value k. $a < 0$: opens down, the vertex is a maximum, value k.

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Card 7 (extension)



The dashed curve is $y = x^2$.

The solid curve has exactly the same shape.

You have this relation only as a graph. Write it as an equation in the form $y = a(x - h)^2 + k$.

Check your equation with two of the marked points.

Card 8 (extension)

In words: the graph of $y = x^2$ is reflected in the x-axis, stretched vertically by a factor of 2, and translated 4 units left and 1 unit up. Write its equation.

Now the other way: describe

$$y = \tfrac{1}{2}(x - 1)^2 - 6$$

in words. Test your words by the step pattern: list the points it gives from the vertex.

Card 9 (extension)

x	y
1	4
2	−2
3	−4
4	−2
5	4

This relation is given only as a table of values.

Which row is the vertex? How can you tell?

Use the steps from the vertex to find a. Then write the equation in vertex form, and check it with a row you did not use.

Horizontal Translations and Vertex Form — Backs

Answers for the cards on the other side of this sheet.

Card 7 (extension) — answer

$y = (x + 2)^2 - 3$.

Same shape and opening up, so $a = 1$. Vertex $(-2, -3)$, so $h = -2$ and $k = -3$: $y = (x - (-2))^2 + (-3)$.

Check by substituting:

$(0, 1)$: $(0 + 2)^2 - 3 = 1$
$(-4, 1)$: $(-4 + 2)^2 - 3 = 1$
$(1, 6)$: $(1 + 2)^2 - 3 = 6$

Card 9 (extension) — answer

Vertex $(3, -4)$: the y-values match in pairs on either side of it, and it is the lowest.

From the vertex, y goes up 2, then up 6 more. The step pattern is $1a, 3a, \ldots$, so $a = 2$.

$y = 2(x - 3)^2 - 4$.

Check $x = 1$: $2(1 - 3)^2 - 4 = 2(4) - 4 = 4$.

Card 8 (extension) — answer

$y = -2(x + 4)^2 + 1$, with vertex $(-4, 1)$.

$y = \frac{1}{2}(x - 1)^2 - 6$ is $y = x^2$ compressed vertically by a factor of $\frac{1}{2}$, then translated 1 unit right and 6 units down. Vertex $(1, -6)$, axis $x = 1$, opens up, minimum value -6.

Steps up 0.5, 1.5, 2.5, 3.5: $(2, -5.5)$, $(3, -4)$, $(4, -1.5)$, $(5, 2)$, and their mirror images at $x = 0, -1, -2, -3$.