Task Cards

Print double-sided (flip on the long edge: Long-Edge binding): three sheets. Card 1 and Extension A are double width. The slot beside Extension B is empty. Cut along the frames.

Card 1

Power	Value
2^5	32
2^4	16
2^3	8
2^2	4
2^1	2
2^0	?
2^{-1}	?
2^{-2}	?
2^{-3}	?

The table shows powers of 2. Each row lowers the exponent by 1.

Copy the table onto your board.

As a group, agree on the four values marked ?.

Card 2

Make the same kind of table for powers of 3.

Start at

$$3^4 = 81$$

and go down one row at a time to 3^{-3}.

Card 3

Work out

$$2^3 \div 2^3$$

in two ways:

- by the exponent law for dividing powers with the same base, and
- by finding the value of 2^3 first.

What does that say about 2^0?

Task Cards — Backs

Answers for the cards on the other side of this sheet.

Card 1 — answer

Power	Value
2^5	32
2^4	16
2^3	8
2^2	4
2^1	2
2^0	1
2^{-1}	$\frac{1}{2}$
2^{-2}	$\frac{1}{4}$
2^{-3}	$\frac{1}{8}$

Each step down **divides by 2**. The pattern carries on past 2^1:

$2 \div 2 = 1$, so $2^0 = 1$.

$2^{-1} = \frac{1}{2}$, $2^{-2} = \frac{1}{4}$, $2^{-3} = \frac{1}{8}$.

Set $2^3 = 8$ beside $2^{-3} = \frac{1}{8}$: the negative exponent gives one over the matching positive power, $\frac{1}{2^3}$.

Card 3 — answer

Law: $2^3 \div 2^3 = 2^{3-3} = 2^0$.

Values: $8 \div 8 = 1$.

Both ways must give the same answer, so $2^0 = 1$, the same as the pattern on Card 1.

Any number (except 0) divided by itself is 1, so the same holds for every base but 0.

Card 2 — answer

Each step down **divides by 3**: 81, 27, 9, 3, then

$3^0 = 1$, $3^{-1} = \frac{1}{3}$, $3^{-2} = \frac{1}{9}$, $3^{-3} = \frac{1}{27}$.

The same as for 2: the power with exponent 0 is 1, and each negative power is one over the matching positive power: $3^{-2} = \frac{1}{9} = \frac{1}{3^2}$.

Different: the step is divide by 3, so the fractions are thirds, ninths, and twenty-sevenths.

Task Cards

Print double-sided (flip on the long edge: Long-Edge binding): three sheets. Card 1 and Extension A are double width. The slot beside Extension B is empty. Cut along the frames.

Card 4

Work out

$$2^2 \div 2^5$$

in two ways:

- by the exponent law, and
- by writing each power as $2 \times 2 \times \cdots$ and cancelling.

What must 2^{-3} be?

Card 5

Sam says

$$2^{-3} = -8.$$

Show Sam why this is wrong.

Card 6

Jordan says

$$5^0 = 0$$

"because zero fives multiplied together is nothing."

Show Jordan why this is wrong.

Card 7

Find the value of each one. Be ready to explain each one to another group.

$$(-3)^{-2} \qquad \left(\tfrac{1}{2}\right)^{-1}$$

$$10^{-3} \qquad 4^0 \times 4^{-1}$$

Task Cards — Backs

Answers for the cards on the other side of this sheet.

Card 5 — answer

$2^{-3} = \frac{1}{8}$, which is positive.

Pattern: 8, 4, 2, 1, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$. Halving a positive number never gives a negative one.

Law: $2^{-3} = 2^2 \div 2^5 = 4 \div 32 = \frac{1}{8}$.

A negative exponent means **one over** the power with the positive exponent. It does not make the number negative.

Card 4 — answer

Law: $2^2 \div 2^5 = 2^{2-5} = 2^{-3}$.

Cancelling:

$$\frac{2 \times 2}{2 \times 2 \times 2 \times 2 \times 2} = \frac{1}{2 \times 2 \times 2} = \frac{1}{2^3} = \frac{1}{8}.$$

So $2^{-3} = \dfrac{1}{2^3} = \dfrac{1}{8}$, the same value as in the table on Card 1.

The three 2s left on the bottom are the 3 in 2^{-3}.

Card 7 — answer

$$(-3)^{-2} = \frac{1}{(-3)^2} = \frac{1}{9}$$

$$\left(\tfrac{1}{2}\right)^{-1} = \frac{1}{\frac{1}{2}} = 2$$

$$10^{-3} = \frac{1}{10^3} = \frac{1}{1000} = 0.001$$

$4^0 \times 4^{-1} = 1 \times \frac{1}{4} = \frac{1}{4}$ (or by the law, $4^{0+(-1)} = 4^{-1} = \frac{1}{4}$)

Card 6 — answer

$5^0 = 1$.

Pattern: 125, 25, 5, each found by dividing by 5. One more step gives $5 \div 5 = 1$.

Law: $5^2 \div 5^2 = 5^{2-2} = 5^0$, and $25 \div 25 = 1$.

Jordan's error: a power multiplies factors onto a start of 1, not 0. $5^3 = 1 \times 5 \times 5 \times 5$. With zero fives, the 1 is all that is left.

Task Cards

Print double-sided (flip on the long edge: Long-Edge binding): three sheets. Card 1 and Extension A are double width. The slot beside Extension B is empty. Cut along the frames.

Extension A

$y = x^2$

x	y
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9

$y = 2^x$

x	y
-3	$\frac{1}{8}$
-2	$\frac{1}{4}$
-1	$\frac{1}{2}$
0	1
1	2
2	4
3	8

For groups that finish early.

The tables show two relations.

Is $y = 2^x$ a quadratic relation? How do you know?

Extension B

For groups that finish early.

At noon, a blood test shows 24 mg of a medicine in a patient's body. The amount in the body halves every hour.

Let t be the number of hours after noon, so 11 a.m. is $t = -1$.

How much medicine was in the body at 9 a.m.?

Task Cards — Backs

Answers for the cards on the other side of this sheet.

Extension A — answer

No.

$y = x^2$: first differences $-5,\ -3,\ -1,\ 1,\ 3,\ 5$; second differences $2,\ 2,\ 2,\ 2,\ 2$.

$y = 2^x$: first differences $\frac{1}{8},\ \frac{1}{4},\ \frac{1}{2},\ 1,\ 2,\ 4$; second differences $\frac{1}{8},\ \frac{1}{4},\ \frac{1}{2},\ 1,\ 2$.

The second differences of $y = x^2$ are constant, so it is quadratic. The second differences of $y = 2^x$ are not constant (they double), so it is not quadratic. Also, the table of $y = x^2$ is symmetric about $x = 0$, and the table of $y = 2^x$ is not.

$y = 2^x$ doubles each time x goes up by 1. It is called an **exponential** relation.

Extension B — answer

Going back one hour **doubles** the amount: 11 a.m. ($t = -1$): 48 mg; 10 a.m. ($t = -2$): 96 mg; 9 a.m. ($t = -3$): **192 mg**.

As an equation: $A = 24 \times \left(\frac{1}{2}\right)^t$. At $t = -3$: $24 \times \left(\frac{1}{2}\right)^{-3} = 24 \times 8 = 192$.