Quadratic Relations and Finite Differences Task Cards

Print double-sided (flip on the long edge: Long-Edge binding): three sheets. Cards 1, 5, and 6 are double width. Sheet 3 holds Card 6 only, in its top row. Cut along the frames.

Card 1

$y = x^2$	for $x = -3, -2, -1, 0, 1, 2, 3$
$y = -x^2$	for $x = -3, -2, -1, 0, 1, 2, 3$
$y = x^2 - 4x$	for $x = -1, 0, 1, 2, 3, 4, 5$
$y = x^2 + 2x - 3$	for $x = -4, -3, -2, -1, 0, 1, 2$

Here are four relations, each with the x values to use.

On your whiteboard, make a table of values for each relation.

Then graph each relation on its own grid, and label each graph with its equation.

Card 2

Look at the four graphs on your group's whiteboard.

List everything they have in common.

Then list everything that differs from one graph to another.

Last, choose one graph and describe it in words alone, so that someone who cannot see it could sketch it.

Card 3

Each relation is now on your group's whiteboard as an equation, as a table of values, and as a graph.

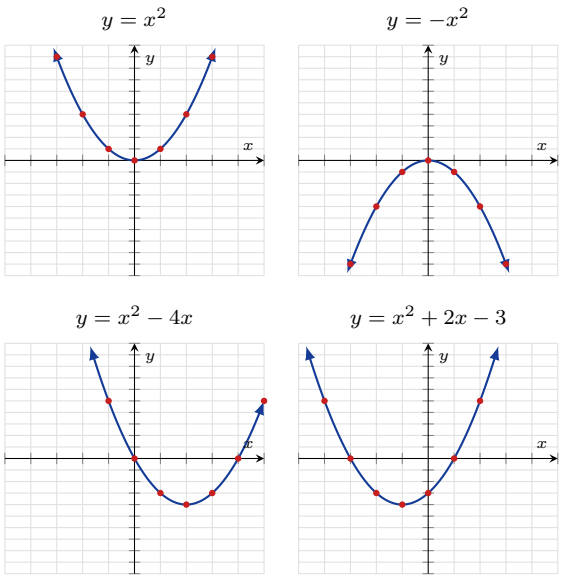
Where does each graph cross the y-axis?

Find a way to read that from the equation alone, without a table or a graph.

Quadratic Relations and Finite Differences Task Cards — Backs

Answers for the cards on the other side of this sheet.

Card 1 — answer



$y = x^2$

x	y
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9

$y = -x^2$

x	y
-3	-9
-2	-4
-1	-1
0	0
1	-1
2	-4
3	-9

$y = x^2 - 4x$

x	y
-1	5
0	0
1	-3
2	-4
3	-3
4	0
5	5

$y = x^2 + 2x - 3$

x	y
-4	5
-3	0
-2	-3
-1	-4
0	-3
1	0
2	5

Card 3 — answer

$y = x^2$, $y = -x^2$, and $y = x^2 - 4x$ all cross at 0; $y = x^2 + 2x - 3$ crosses at -3.

A graph crosses the y-axis where $x = 0$. Put $x = 0$ into the equation: every term with an x in it becomes 0, and only the term without an x (the **constant term**) is left. That number is the **y-intercept**.

A relation can be described in words, as a table of values, as an equation, and as a graph. Here you read a feature of the graph straight from the equation.

Card 2 — answer

In common: each is a U-shaped curve (one is upside down); each has one turning point; each has a vertical mirror line through the turning point; each crosses the y-axis exactly once.

Differs: three open up and one ($y = -x^2$) opens down; the turning points are in different places: $(0, 0)$ for $y = x^2$ and $y = -x^2$, $(2, -4)$ for $y = x^2 - 4x$, $(-1, -4)$ for $y = x^2 + 2x - 3$.

Words: the curve is a **parabola**; the turning point is the **vertex**; the mirror line is the **axis of symmetry**; the vertex is a **minimum** when it opens up and a **maximum** when it opens down; where it crosses the y-axis is the **y-intercept**.

In words, for $y = x^2 - 4x$: it opens up, its vertex is $(2, -4)$, its axis of symmetry is $x = 2$, and it crosses the x-axis at 0 and 4.

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Card 4

Go back to your four tables from Card 1.

In each table, find the difference between each pair of consecutive y values.

Then find the differences between those differences.

What do you notice?

Card 7

Go back to the class table of thumb lengths and palm areas from Day 1.

Could the relation between thumb length and palm area be quadratic?

What would you need to check, and why?

Why can you not check it with the class data as we collected it?

Card 5

$y = 4x - 3$
a **linear** relation

x	y	First differences	Second differences
0	-3		
		$1 - (-3) = 4$	
1	1		$4 - 4 = 0$
		$5 - 1 = 4$	
2	5		$4 - 4 = 0$
		$9 - 5 = 4$	
3	9		$4 - 4 = 0$
		$13 - 9 = 4$	
4	13		$4 - 4 = 0$
		$17 - 13 = 4$	
5	17		

Each difference is the lower value minus the one above it.

$y = 3x^2 - x + 2$
a **quadratic** relation

x	y	First differences	Second differences
0	2		
		$4 - 2 = 2$	
1	4		$8 - 2 = 6$
		$12 - 4 = 8$	
2	12		$14 - 8 = 6$
		$26 - 12 = 14$	
3	26		$20 - 14 = 6$
		$46 - 26 = 20$	
4	46		$26 - 20 = 6$
		$72 - 46 = 26$	
5	72		

Each difference is the lower value minus the one above it.

A relation whose highest power of x is x^2 is called **quadratic**.

Write a rule a classmate could use to tell a linear relation from a quadratic relation, using only a table of values.

Test your rule on your tables from Card 1.

Quadratic Relations and Finite Differences Task Cards — Backs

Answers for the cards on the other side of this sheet.

Card 7 — answer

To check, we need second differences, and those need x values in equal steps.

Our thumb lengths are whatever our thumbs happen to be (5.3, 5.5, 6.0, 6.2, … cm), not evenly spaced, and each measurement carries some error, so differences of the raw data would not settle it.

Instead: read palm areas off the curve of best fit at evenly spaced thumb lengths (5, 5.5, 6, … cm), then test the second differences. The OTL returns to this.

Card 4 — answer

The first column of differences changes each time, by the same amount; the second column is constant.

$y = x^2$, $y = x^2 - 4x$, and $y = x^2 + 2x - 3$: $-5, -3, -1, 1, 3, 5$; then 2 every time.
$y = -x^2$: $5, 3, 1, -1, -3, -5$; then -2 every time.

These are the **first differences** and the **second differences**.

Card 5 — answer

Linear: the first differences are constant (so the second differences are all 0).

Quadratic: the first differences are not constant, but the second differences are constant (and not 0).

Neither: neither column is constant.

Only with equal steps: differences only mean something when the x values go up by the same step every row. Both tables here step by 1, and so do your tables from Card 1.

All four relations from Card 1 pass the quadratic test.

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Card 6

Priya tested the relation $y = x^2 - 2x - 3$.

x	y	First differences	Second differences
-3	12		
		-12	
-1	0		9
		-3	
0	-3		3
		0	
2	-3		3
		3	
3	0		9
		12	
5	12		

"The second differences are not all the same, so this relation is not quadratic."

Adapted from Principles of Mathematics 10, McGraw-Hill Ryerson, 2007.

Priya says this relation is not quadratic.

Do you agree?

Her arithmetic is correct. Find the flaw in her reasoning, if there is one.

Quadratic Relations and Finite Differences Task Cards — Backs

Answers for the cards on the other side of this sheet.

Card 6 — answer

x	y	First differences	Second differences
-3	12		
		-7	
-2	5		2
		-5	
-1	0		2
		-3	
0	-3		2
		-1	
1	-4		2
		1	
2	-3		2
		3	
3	0		2
		5	
4	5		2
		7	
5	12		

Disagree. Her x values are not evenly spaced (steps of $2, 1, 2, 1, 2$), so her differences compare jumps over different distances.

With x going up in equal steps (left), the second differences are all 2: the relation is **quadratic**.

The differences test only works when the x values are **evenly spaced**.