Stretches, Compressions, and Reflections

Print double-sided (flip on the long edge: Long-Edge binding): three sheets. Cards 5 and 7 are double width. Cut along the frames.

Card 1

On your board, draw $y = x^2$ again on one grid, for x from -3 to 3.

Make a table of values for

$$y = 2x^2$$

for x from -2 to 2, and plot it on a second grid, beside the first.

At the same x, how does the y-value of $y = 2x^2$ compare with the y-value of $y = x^2$? What does that do to the graph? What stays the same?

Card 2

On a third grid, make a table of values and plot

$$y = \frac{1}{2}x^2$$

for x from -4 to 4. Put it beside your other two graphs.

How does this graph compare with $y = x^2$? With $y = 2x^2$?

Predict what $y = \frac{1}{4}x^2$ would look like, and say why.

Card 3

Make tables of values for

$$y = -x^2 \text{ for } x \text{ from } -3 \text{ to } 3,$$

$$y = -2x^2 \text{ for } x \text{ from } -2 \text{ to } 2.$$

Plot $y = -x^2$ on your fourth grid, beside your graph of $y = x^2$. Compare your table for $y = -2x^2$ with your table for $y = 2x^2$.

Each new relation looks like one you met on Cards 1 and 2. How are they related?

What does the negative sign do?

Card 4

You have now met

$$y = ax^2$$

for $a = 2$, $\frac{1}{2}$, -1, and -2.

In words, how does the number a change the graph of $y = x^2$? Sort what you found into cases.

Which values of a make the graph narrower? Wider? Turn it upside down?

What does a leave alone?

Stretches, Compressions, and Reflections — Backs

Answers for the cards on the other side of this sheet.

Card 2 — answer

x	y
-4	8.0
-3	4.5
-2	2.0
-1	0.5
0	0.0
1	0.5
2	2.0
3	4.5
4	8.0

Every y-value is **half** the y-value of $y = x^2$, so every point is half as far from the x-axis. The graph is squashed toward the x-axis: wider and flatter.

Same vertex $(0, 0)$, axis $x = 0$, and it opens up.

$y = \frac{1}{4}x^2$: every y a quarter of $y = x^2$, so wider still (at $x = 4$, $y = 4$).

Card 1 — answer

x	y
-2	8
-1	2
0	0
1	2
2	8

Every y-value is **doubled**, so every point is twice as far from the x-axis. The graph is pulled away from the x-axis: it looks narrower and steeper.

Same: the vertex $(0, 0)$, the axis of symmetry $x = 0$, and it opens up.

$x = \pm 3$ would give $2 \times 9 = 18$, off the grid.

Card 4 — answer

$a > 1$ or $a < -1$: a **vertical stretch** by a factor of $|a|$ (the size of a, ignoring its sign); the graph is narrower.

$0 < a < 1$ or $-1 < a < 0$: a **vertical compression** by a factor of $|a|$; the graph is wider.

$a < 0$: a **reflection in the x-axis**; the graph opens down, and the vertex is a maximum.

$y = -2x^2$ is both stretched and reflected; $y = -\frac{1}{3}x^2$ is compressed and reflected.

a leaves the vertex $(0, 0)$ and the axis of symmetry $x = 0$ alone.

Card 3 — answer

x	y
-3	-9
-2	-4
-1	-1
0	0
1	-1
2	-4
3	-9

x	y
-2	-8
-1	-2
0	0
1	-2
2	-8

Every y-value is the **opposite** of the one for $y = x^2$ (or $y = 2x^2$), so every point flips to the other side of the x-axis. Both open **down**, and the vertex $(0, 0)$ is now the highest point.

$y = -2x^2$ is the flip of $y = 2x^2$: just as narrow.

Stretches, Compressions, and Reflections

Print double-sided (flip on the long edge: Long-Edge binding): three sheets. Cards 5 and 7 are double width. Cut along the frames.

Card 5

$y = x^2$

x	y	First differences
0	0	
		1
1	1	
		3
2	4	
		5
3	9	

$y = 2x^2$

x	y	First differences
0	0	
		2
1	2	
		6
2	8	
		10
3	18	

$y = 3x^2$

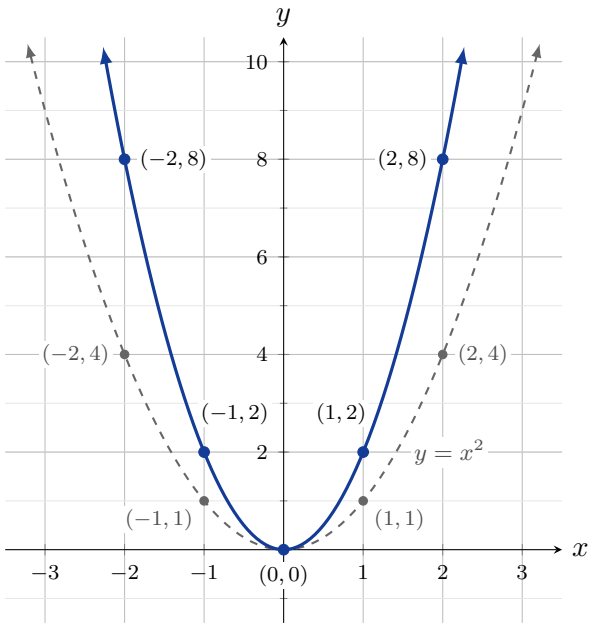
x	y	First differences
0	0	
		3
1	3	
		9
2	12	
		15
3	27	

These tables start at the vertex and step to the right, one unit at a time.

Compare the first differences of the three relations.

What pattern lets you sketch $y = ax^2$ without making a table?

Card 7



The dashed curve is $y = x^2$.

The solid parabola is $y = ax^2$ and also has its vertex at the origin.

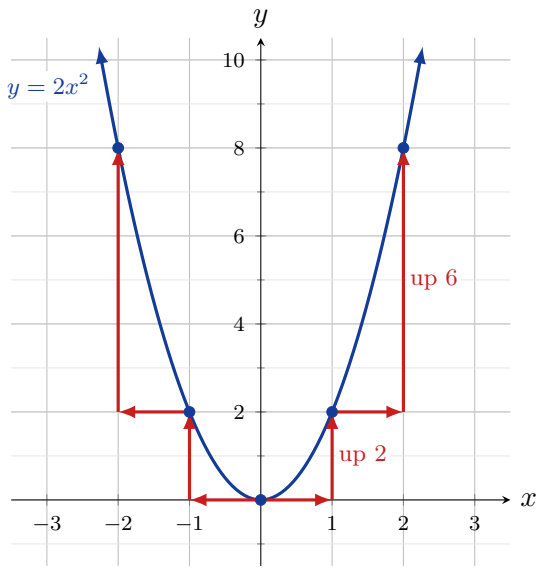
Try to write its equation by identifying the value for a.

How many ways can you find a? Check your equation without plotting anything.

Stretches, Compressions, and Reflections — Backs

Answers for the cards on the other side of this sheet.

Card 5 — answer



The first differences of $y = ax^2$, from the vertex, are a, $3a$, $5a$, $\ldots$:

$y = x^2$: 1, 3, 5
$y = 2x^2$: 2, 6, 10
$y = 3x^2$: 3, 9, 15

The step pattern: start at the vertex. Go over 1 and up a, over 1 and up $3a$, over 1 and up $5a$. Do the same on the other side, by symmetry.

When $a < 0$, each step goes down.

Card 7 — answer

$y = 2x^2$.

By the step pattern: from $(0, 0)$, over 1 and up 2, so $a = 2$; the next step is up 6, which is $3a$.

By substitution: $(1, 2)$ in $y = ax^2$ gives $2 = a(1)^2$, so $a = 2$.

By comparing with $y = x^2$: at the same x, each y on the solid curve is double the y on the dashed one: 1 to 2 and 4 to 8, so $a = 2$.

Check the other points:

$(2, 8)$: $2(2)^2 = 8$
$(-1, 2)$: $2(-1)^2 = 2$
$(-2, 8)$: $2(-2)^2 = 8$

Stretches, Compressions, and Reflections

Print double-sided (flip on the long edge: Long-Edge binding): three sheets. Cards 5 and 7 are double width. Cut along the frames.

Card 6

Use only the pattern from Card 5, with no table and no grid. List the points your steps give for

$$y = 3x^2$$

Then write the equation of $y = x^2$ compressed vertically by a factor of $\frac{1}{2}$ and reflected in the x-axis, and list the points your steps give for it.

Stop stepping when the next point would leave a -10 to 10 grid.

Card 8

How are the graphs of

$$y = 2x^2 \quad \text{and} \quad y = -2x^2$$

alike? How are they different?

A classmate says that $y = -2x^2$ must be wider than $y = 2x^2$, because -2 is less than 2.

What would you tell them?

Stretches, Compressions, and Reflections — Backs

Answers for the cards on the other side of this sheet.

Card 8 — answer

Alike: the vertex $(0, 0)$, the axis of symmetry $x = 0$, and the same shape and width: steps of 2, 6, 10 for both, up for one and down for the other.

Different: $y = 2x^2$ opens up, with a minimum of 0; $y = -2x^2$ opens down, with a maximum of 0. Each point (x, y) on one matches $(x, -y)$ on the other: they are reflections of each other in the x-axis.

The classmate: the width depends on how far a is from 0, not on its sign. At $x = 1$, both are 2 units from the x-axis, so neither is wider.

Card 6 — answer

$\boldsymbol{y = 3x^2}$: $(0, 0)$, then over 1 up 3 to $(\pm 1, 3)$. The next step, up 9, would reach $(\pm 2, 12)$, off the grid, so the curve leaves the top of the grid near $x = \pm 1.8$.

$\boldsymbol{y = -\frac{1}{2}x^2}$: steps down 0.5, 1.5, 2.5, 3.5, to $(\pm 1, -0.5)$, $(\pm 2, -2)$, $(\pm 3, -4.5)$, $(\pm 4, -8)$.

Check one point: $-\frac{1}{2}(4)^2 = -8$.