The Parent Parabola and Vertical Translations

Print double-sided (flip on the long edge: Long-Edge binding): three sheets. Cards 1, 5, and 7 are double width. The slot beside Card 8 is empty. Cut along the frames.

Card 1

$y = x^2$

x	y
-3	9
-2	
-1	1
0	
1	
2	4
3	

On your board, beside your grid (not on the axes), copy this table of values for $y = x^2$ and complete it.

Then plot the points from your table and join them with a smooth curve. Keep this grid for $y = x^2$ alone: you will put new graphs beside it.

Mark the lowest point.

What symmetry do you see? Where is the mirror line?

Card 2

On a second grid, make a table of values and plot

$$y = x^2 + 1$$

for the same values of x.

Put it beside your graph of $y = x^2$ and compare.

What changed? What stayed the same?

Card 3

On a third grid, make a table of values and plot

$$y = x^2 - 4$$

for the same values of x.

Look at your three grids side by side. How does this graph sit compared with the other two?

What is the same about all three?

The Parent Parabola and Vertical Translations — Backs

Answers for the cards on the other side of this sheet.

Card 1 — answer

$y = x^2$

x	y
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9

Lowest point, the **vertex**: $(0, 0)$.

The two halves are mirror images in the y-axis, the **axis of symmetry** $x = 0$: x and $-x$ give the same y, such as $(-3, 9)$ and $(3, 9)$.

Card 3 — answer

x	y
-3	5
-2	0
-1	-3
0	-4
1	-3
2	0
3	5

Every point is 4 units lower than on $y = x^2$. The vertex is $(0, -4)$.

All three have the same shape, the axis of symmetry $x = 0$, and open up. Only the up-and-down position differs.

You now have $y = x^2 - 4$ in words, as a table of values, as an equation, and as a graph.

Card 2 — answer

x	y
-3	10
-2	5
-1	2
0	1
1	2
2	5
3	10

Changed: every y is 1 more, so every point is 1 unit higher. The vertex is now $(0, 1)$.

Same: the shape, the axis of symmetry $x = 0$, and it still opens up.

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Card 4

No table this time. You have each relation only as an equation:

$$y = x^2 + 5 \qquad y = x^2 - 2$$

In words, describe where each graph sits compared with $y = x^2$. What do you know about each one before you plot a single point?

Then sketch $y = x^2 - 2$ on your fourth grid to test your words.

Card 6

Think about

$$y = x^2 + k$$

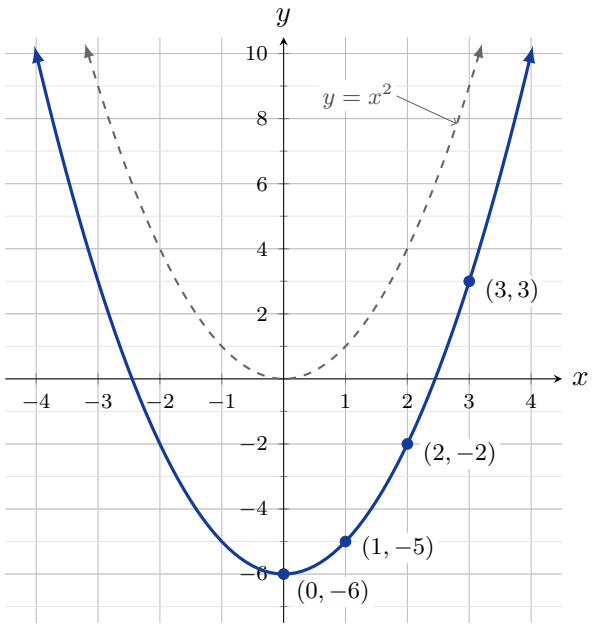
for any number k.

Where does the graph cross the y-axis?

For which values of k does the graph cross the x-axis? Only touch it? Miss it completely?

When it crosses, where?

Card 5



The dashed curve is $y = x^2$.

The solid curve has exactly the same shape.

You have this relation only as a graph. Write it as an equation.

How could you check your equation without plotting anything?

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Answers for the cards on the other side of this sheet.

Card 6 — answer

***y*-intercept:** $(0, k)$, which is also the vertex.

$k > 0$: the vertex is above the x-axis and the graph opens up, so it **never** meets the x-axis.

$k = 0$: it **touches** the x-axis once, at the origin.

$k < 0$: it **crosses twice**, where $x^2 = -k$, at $x = \pm\sqrt{-k}$.

For example, $k = -9$: $x^2 - 9 = 0$, so $x^2 = 9$ and $x = \pm 3$: the points $(-3, 0)$ and $(3, 0)$.

Card 4 — answer

$y = x^2 + 5$ is $y = x^2$ moved 5 units up: vertex $(0, 5)$.

$y = x^2 - 2$ is $y = x^2$ moved 2 units down: vertex $(0, -2)$.

Both open up, have the same shape as $y = x^2$, and have the axis of symmetry $x = 0$.

In general, $y = x^2 + k$ is $y = x^2$ moved k units up (down when k is negative), with vertex $(0, k)$: a **vertical translation**.

On a -10 to 10 grid, $y = x^2 - 2$ fits for x from -3 to 3 ($x = \pm 3$ gives 7); only the points of $y = x^2 + 5$ with x from -2 to 2 would fit ($x = \pm 3$ gives 14).

Card 5 — answer

$y = x^2 - 6$: the vertex moved from $(0, 0)$ to $(0, -6)$, so every point is 6 units lower.

Check by substituting each labelled point:

$(0, -6)$: $0^2 - 6 = -6$
$(1, -5)$: $1^2 - 6 = -5$
$(2, -2)$: $2^2 - 6 = -2$
$(3, 3)$: $3^2 - 6 = 3$

Every point fits, so the equation fits the graph.

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Card 7

$y = x^2 + 5$

x	y	First differences	Second differences
-3	14		
		$9 - 14 = -5$	
-2	9		$-3 - (-5) = 2$
		$6 - 9 = -3$	
-1	6		$-1 - (-3) = 2$
		$5 - 6 = -1$	
0	5		$1 - (-1) = 2$
		$6 - 5 = 1$	
1	6		$3 - 1 = 2$
		$9 - 6 = 3$	
2	9		$5 - 3 = 2$
		$14 - 9 = 5$	
3	14		

Each difference is the lower value minus the one above it.

Here is a table of values for $y = x^2 + 5$, with its first and second differences.

Work out the first and second differences for $y = x^2$, using your table from Card 1.

Compare the two sets of differences.

What does adding 5 change? What does it leave alone?

Card 8

Think back to the rolling can.

Your plot of distance against time looked like half of a parabola, with its vertex at the origin.

Which of today's relations has its vertex at the origin?

Why might the can's graph still not be the graph of that relation?

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Answers for the cards on the other side of this sheet.

Card 7 — answer

$y = x^2$

x	y	First difference	Second difference
-3	9		
		-5	
-2	4		2
		-3	
-1	1		2
		-1	
0	0		2
		1	
1	1		2
		3	
2	4		2
		5	
3	9		

The first differences are the same: -5, -3, -1, 1, 3, 5.

The second differences are the same: 2 every time.

Adding 5 makes every y-value 5 more. Each difference subtracts two y-values, so the 5 cancels.

k changes **where** the graph is (the y-values, the vertex), not its **shape** (the differences).

Card 8 — answer

$y = x^2$ (the case $k = 0$) has its vertex at $(0, 0)$.

But the can's graph need not match it: a can that rolled 100 cm in about 1.8 s does not fit $y = x^2$, since 1.8^2 is only about 3.

The can's curve is the same kind of shape with a different steepness (in centimetres, much steeper). k only slides a graph up or down, so it cannot fix that.