Warmup: Comparing Differences

Adapted from Principles of Mathematics 10, McGraw-Hill Ryerson, 2007.

Work alone or with a partner. Each table below is a relation you can already graph. Write each difference in the row between the two values it comes from.

$y = x^2$

x	y	First differences	Second differences
-3	9		
		$4 - 9 = -5$	
-2	4		$-3 - (-5) = 2$
		$1 - 4 = -3$	
-1	1		_____
		_____	
0	0		_____
		_____	
1	1		_____
		_____	
2	4		_____
		_____	
3	9		

Each difference is the lower value minus the one above it.

$y = 2x^2$

x	y	First differences	Second differences
-3	18		
-2	8		
-1	2		
0	0		
1	2		
2	8		
3	18		

$y = \frac{1}{2}x^2$

x	y	First differences	Second differences
-3	4.5		
-2	2.0		
-1	0.5		
0	0.0		
1	0.5		
2	2.0		
3	4.5		

1. **Differences.** Fill in the first differences and then the second differences for all three tables. Take care with the signs: each difference is the lower y-value minus the one above it.

2. **Recall.** Is each relation linear, quadratic, or neither? How do your differences tell you?

3. **Notice.** Look across the three tables. Write one thing you notice about the first differences and one thing you notice about the second differences.

Warmup: Comparing Differences — Answers

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The differences are filled in below. Answers in words are model sentences; other things noticed that are true are fine.

$y = x^2$

x	y	First differences	Second differences
-3	9		
		$4-9=-5$	
-2	4		$-3-(-5)=2$
		$1-4=-3$	
-1	1		$-1-(-3)=2$
		$0-1=-1$	
0	0		$1-(-1)=2$
		$1-0=1$	
1	1		$3-1=2$
		$4-1=3$	
2	4		$5-3=2$
		$9-4=5$	
3	9		

Each difference is the lower value minus the one above it.

$y = 2x^2$

x	y	First differences	Second differences
-3	18		
		-10	
-2	8		4
		-6	
-1	2		4
		-2	
0	0		4
		2	
1	2		4
		6	
2	8		4
		10	
3	18		

$y = \frac{1}{2}x^2$

x	y	First differences	Second differences
-3	4.5		
		-2.5	
-2	2.0		1
		-1.5	
-1	0.5		1
		-0.5	
0	0.0		1
		0.5	
1	0.5		1
		1.5	
2	2.0		1
		2.5	
3	4.5		

1. **Differences.** Fill in the first differences and then the second differences for all three tables. Take care with the signs: each difference is the lower y-value minus the one above it.

   $y = x^2$: first $-5, -3, -1, 1, 3, 5$; second all 2.
   $y = 2x^2$: first $-10, -6, -2, 2, 6, 10$; second all 4.
   $y = \frac{1}{2}x^2$: first $-2.5, -1.5, -0.5, 0.5, 1.5, 2.5$; second all 1.

2. **Recall.** Is each relation linear, quadratic, or neither? How do your differences tell you?

   All three are quadratic. The first differences are not constant, so none is linear, but the second differences are constant (2, 4, and 1).

3. **Notice.** Look across the three tables. Write one thing you notice about the first differences and one thing you notice about the second differences.

   Some true things to notice: the first differences of $y = 2x^2$ are double those of $y = x^2$, and those of $y = \frac{1}{2}x^2$ are half. In every table the first differences change sign at $x = 0$ (the vertex) and match in size on either side. The second differences are 2, 4, and 1: each is twice the number in front of x^2.

   Collect what students notice, but do not draw the conclusion at the desks: Cards 1 to 4 find the effect of the number in front of x^2 on the graph, and Card 5 turns these first differences into the step pattern.