Warmup: Linear, Quadratic, or Neither

Adapted from Principles of Mathematics 10, McGraw-Hill Ryerson, 2007.

Work alone or with a partner. Each table below shows a relation between x and y. Write each difference in the row between the two values it comes from.

Table A

x	y	First differences
0	3	
1	7	$7 - 3 = 4$
2	11	
3	15	
4	19	
5	23	

Each difference is the lower value minus the one above it.

Table B

x	y	First differences	Differences of the differences
0	4		
1	7		
2	14		
3	25		
4	40		
5	59		

Table C

x	y	First differences	Differences of the differences
0	3		
1	6		
2	12		
3	24		
4	48		
5	96		

1. **Recall.** From Grade 9: what do the first differences in a table tell you about a relation? Then fill in the First differences column for all three tables.

2. **Linear.** Which table shows a linear relation? How do you know?

3. **Not linear.** Tables B and C are both not linear. Are they the same kind of "not linear"? Fill in the Differences of the differences column for Tables B and C, then say what you notice.

Warmup: Linear, Quadratic, or Neither — Answers

Adapted from Principles of Mathematics 10, McGraw-Hill Ryerson, 2007.

The differences are filled in below. Answers in words are model sentences; other answers that are well argued are fine.

Table A

x	y	First differences
0	3	
		$7-3=4$
1	7	
		$11-7=4$
2	11	
		$15-11=4$
3	15	
		$19-15=4$
4	19	
		$23-19=4$
5	23	

Each difference is the lower value minus the one above it.

Table B

x	y	First differences	Differences of the differences
0	4		
		3	
1	7		4
		7	
2	14		4
		11	
3	25		4
		15	
4	40		4
		19	
5	59		

Table C

x	y	First differences	Differences of the differences
0	3		
		3	
1	6		3
		6	
2	12		6
		12	
3	24		12
		24	
4	48		24
		48	
5	96		

1. **Recall.** From Grade 9: what do the first differences in a table tell you about a relation? Then fill in the First differences column for all three tables.

   First differences show how much y changes each time x goes up by the same step (here, by 1). When they are all the same, y changes at a constant rate, so the relation is linear, and that constant difference is the slope.

   Table A: $4, 4, 4, 4, 4$. Table B: $3, 7, 11, 15, 19$. Table C: $3, 6, 12, 24, 48$.

2. **Linear.** Which table shows a linear relation? How do you know?

   Table A. Its first differences are all 4, so y goes up by 4 every time x goes up by 1. (Its equation is $y = 4x + 3$.) Tables B and C have first differences that keep changing, so they are not linear.

3. **Not linear.** Tables B and C are both not linear. Are they the same kind of "not linear"? Fill in the Differences of the differences column for Tables B and C, then say what you notice.

   Table B: $4, 4, 4, 4$. Table C: $3, 6, 12, 24$.

   No, they are different kinds. In Table B the differences of the differences are constant (all 4), even though the first differences are not. In Table C they are not constant: each column just doubles again, like the y values themselves, so taking differences never settles down.

   The differences of the differences are called second differences. The word for a relation like Table B's, *quadratic*, comes in today's Discussion; do not give it away at the desks. (Table B is $y = 2x^2 + x + 4$; Table C is $y = 3 \times 2^x$.)