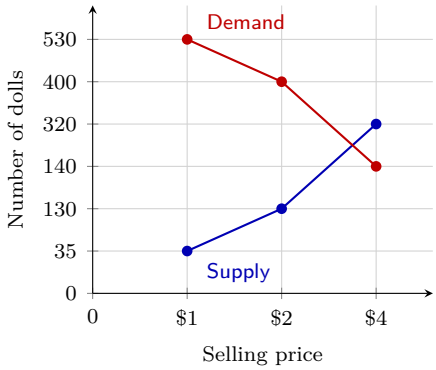
Warmup: Supply and Demand Debrief

Take out your work for *Linear Systems: Supply and Demand Applications* (the BurgerRama dolls). Work through each check with a partner. Make any corrections on your own work in a different colour.

1. **Check your scale.** Phillipe plotted the six points from the table correctly on the graph shown here, but the points for each line do not fall in a straight line.

   a) What went wrong with Phillipe's axes? Explain why this bends the lines.

   b) Now look at your own graph. On each axis, does every square stand for the same amount? Do both axes start at 0, so that you can see where each line meets an axis? Fix your scale if you need to.



2. **Check your equations.** You used two of the three points in the table to find each equation.

   a) Substitute the point you did *not* use into each equation. Does it satisfy the equation? If not, look for the error in your slope or in your value of b.

   b) One of your slopes should be positive and the other negative. Which is which? Explain why that makes sense for doll buyers and doll suppliers.

3. **Check your graph readings.** Questions 3a and 3b of the task each ask about a *gap* between the two lines, not a single reading.

   a) Substitute $p = 2.5$ into both of your equations, then subtract. How close was your graph estimate for 3a? Do the same with $p = 3.8$ for 3b.

   b) At which of these two prices is the demand line above the supply line? What does that mean for the stores?

4. **Check your intercepts.** Questions 3c and 3d of the task each ask about a point where a line meets an axis.

   a) For each question, which line did you use, and which axis does it meet? Use your equations to calculate both values. How close were your graph estimates?

   b) In the supply equation, b is negative. What would that number mean in this situation? Why is it not the answer to 3d?

5. **Check your solution.** In question 6 of the task, you solved the linear system.

   a) Substitute your values of p and n into *both* equations. Is each equation satisfied? Does your solution agree with your estimate from the graph in 3e?

   b) Read your concluding sentence. Does it give both the price and the number of dolls, with units, and say what they mean for BurgerRama?

Warmup: Supply and Demand Debrief — Final Answers

Final answers only. Readings taken from a graph are estimates, so answers to question 3 of the task that are close to these are fine.

1. Check your scale.

a) Phillipe wrote the numbers from the table along each axis and spaced them evenly. Equal distances on Phillipe's axes do not stand for equal amounts: one square up is 95 dolls in one place (35 to 130) but only 10 dolls in another (130 to 140), and the price axis skips \$3. The data are still linear; the uneven scale bends the lines.

b) Corrections will vary. A scale that works well: price from \$0 to \$6 (for example, \$0.50 or \$0.20 for each square) and dolls from 0 to 700 (for example, 50 or 20 dolls for each square), starting at 0 on both axes.



2. Check your equations.

a) Supply: $n = 95p - 60$. Demand: $n = -130p + 660$. Each equation is satisfied by all three of its points; for example, $95(4) - 60 = 320$ and $-130(4) + 660 = 140$.

b) The supply slope is positive (95): at a higher price, more dolls are supplied. The demand slope is negative (-130): at a higher price, fewer customers want a doll.

3. Check your graph readings.

a) At \$2.50: demand is 335 and supply is 177.5, so about 158 disappointed customers.
At \$3.80: supply is 301 and demand is 166, so 135 unsold dolls.

b) At \$2.50 the demand line is above the supply line: more dolls are wanted than supplied (a shortage). At \$3.80 the supply line is above: there are dolls left over (a surplus).

4. Check your intercepts.

a) 3c uses the demand line where it meets the vertical axis ($p = 0$): $n = 660$ dolls.
3d uses the supply line where it meets the horizontal axis ($n = 0$): $0 = 95p - 60$, so $p = \dfrac{60}{95} \approx \$0.63$.

b) $b = -60$ would mean a supply of -60 dolls at a price of \$0, which is not possible. Question 3d asks for the *price* when the supply is 0, not the supply when the price is 0.

5. Check your solution.

a) $p = 3.2$ and $n = 244$. Check: $95(3.2) - 60 = 244$ and $-130(3.2) + 660 = 244$.

b) At a price of \$3.20 for each doll, supply and demand are equal: each store is supplied with 244 dolls a week and customers ask for 244 dolls a week.

Supply and Demand Applications — Worked Solutions

Each question from the task is followed by a solution. The notes in blue explain *why* each step was taken. Compare these solutions with your own work, and ask about anything that is still unclear.

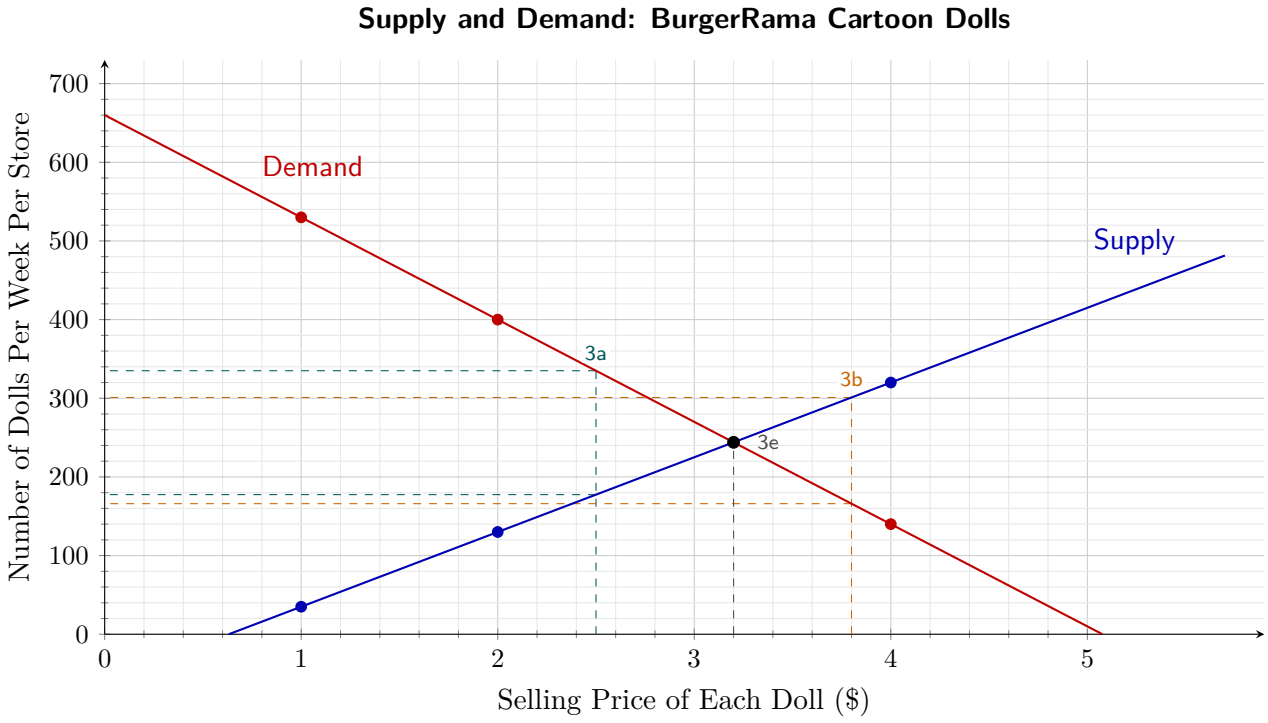
The situation. BurgerRama will sell a cartoon-character doll. The marketing director must choose a selling price. Data from earlier promotions show how many dolls are supplied, and how many are requested, at three different prices.

Selling price of each doll	Number supplied per week per store	Number requested per week per store
\$1.00	35	530
\$2.00	130	400
\$4.00	320	140

Questions 1 and 2

1. Plot points representing selling price and the number of dolls supplied (supply) on a graph, with selling price on the x-axis and number of dolls per week per store on the y-axis. Draw the line through the data points, and write the word "Supply" on this line.

2. Plot points representing selling price and number of dolls requested (demand) on the same graph. Draw the line through these points. Write the word "Demand" on this line.



Why this scale? Each axis starts at 0 and goes up in equal steps (\$1 for every 5 squares; 100 dolls for every 5 squares). Equal distances must stand for equal amounts, or the points will not fall in a straight line. The axes go past the data (up to 700 dolls, and past \$5) because question 3 asks where the lines meet the axes.

Why do three points fall on one line? For supply, a \$1 increase in price (\$1 to \$2) adds 95 dolls, and a \$2 increase (\$2 to \$4) adds 190 dolls, which is twice as many. The rate of change is constant, so the relation is linear. The same is true for demand.

Question 3

Use your graph to answer the following questions.

Answers read from a graph are estimates. Yours may differ a little from these. The dashed lines on the graph show where each reading was taken.

a) *If King sets the price at \$2.50 per doll, how many disappointed customers will each store have during the week?*

Go up from \$2.50 on the price axis to each line, then across to the vertical axis.

At \$2.50, demand is about 335 dolls, but supply is only about 180 dolls.

$335 - 180 = 155$, so there will be about 155 disappointed customers who do not get a doll.

Why subtract? Customers want 335 dolls, but only 180 are available. The disappointed customers are the gap between the two lines at this price.

b) *If King sets the price at \$3.80 per doll, how many unsold dolls will remain at each store at the end of a week?*

At \$3.80, supply is about 300 dolls, but demand is only about 165 dolls.

$300 - 165 = 135$, so about 135 dolls will be unsold.

Why is it the other way around this time? At \$3.80 the supply line is above the demand line, so each store has more dolls than customers want.

c) *According to this graph, if the company could give the dolls away, how many would each store need per week?*

Giving the dolls away means a price of \$0. The demand line meets the vertical axis at about 660, so each store would need about 660 dolls.

Why the demand line? "How many would each store need" asks how many dolls customers want. A price of \$0 is on the vertical axis, so this is the y-intercept of the demand line.

d) *According to this graph, what price would make the doll supply so tight that the average number available to each store would be zero?*

The supply line meets the horizontal axis at about \$0.60, so a supply of 0 dolls occurs at a price of about \$0.60.

Why the horizontal axis? A supply of 0 dolls means the height of the supply line is 0. This is the x-intercept of the supply line.

e) *Estimate the price where supply and demand will be in equilibrium.*

The two lines cross at a price of about \$3.20, so supply and demand are equal (in equilibrium) at about \$3.20.

Why the point where the lines cross? It is the only point that is on both lines, so it is the only price where the number supplied equals the number requested.

Question 4

Use two of the points given to find an equation that relates n, the number of dolls supplied, to p, the price of a doll.

Write the data as ordered pairs: (selling price, number supplied), or (p, n). Use $(1, 35)$ and $(2, 130)$.

Why this order? Price is on the x-axis and number of dolls is on the y-axis, so p takes the place of x and n takes the place of y. Any two of the three points give the same equation.

Step 1: Find the slope.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{130 - 35}{2 - 1} \\ &= 95 \end{aligned}$$

What the slope means: for each \$1 increase in price, 95 more dolls are supplied.

Step 2: Find the y-intercept.

$$\begin{aligned} y &= mx + b \\ 35 &= 95(1) + b \\ 35 - 95 &= b \\ -60 &= b \end{aligned}$$

Why substitute a point? We know m, but not b. The point $(1, 35)$ is on the line, so its coordinates must satisfy the equation. That leaves b as the only unknown.

Step 3: Write the equation.

Replace m and b in $y = mx + b$, then use p in place of x and n in place of y:

$$n = 95p - 60$$

Therefore, the equation that relates supply, n, to price, p, is $n = 95p - 60$.

Check with the point that was not used, $(4, 320)$: $95(4) - 60 = 380 - 60 = 320$. It works, so the equation is correct.

Question 5

Use two of the points given to find an equation that relates n, the number of dolls in demand, to p, the price of a doll.

Write the data as ordered pairs: (selling price, number in demand), or (p, n). Use $(1, 530)$ and $(2, 400)$.

Step 1: Find the slope.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{400 - 530}{2 - 1} \\ &= -130 \end{aligned}$$

Why negative? The demand line goes down from left to right: for each \$1 increase in price, 130 fewer dolls are requested.

Step 2: Find the y-intercept.

$$\begin{aligned} y &= mx + b \\ 400 &= (-130)(2) + b \\ 400 &= -260 + b \\ 400 + 260 &= b \\ 660 &= b \end{aligned}$$

Either point can be used here. This time the point $(2, 400)$ was substituted.

Step 3: Write the equation.

$$n = -130p + 660$$

Therefore, the equation that relates demand, n, to price, p, is $n = -130p + 660$.

Check with the point that was not used, $(4, 140)$: $-130(4) + 660 = -520 + 660 = 140$. Also, $b = 660$ matches the answer to question 3c that was read from the graph.

Question 6

You now have two equations: one that relates supply to price, and another that relates demand to price. Solve this linear system. What price, p, will result in demand and supply, n, being equal?

Ⓐ $n = 95p - 60$ Ⓑ $n = -130p + 660$

Why substitution? Both equations are already solved for n, so the expression for n from one equation can replace n in the other. This leaves one equation with one unknown, p.

Substitute Ⓐ into Ⓑ to find p.

$$\begin{aligned} n &= -130p + 660 \\ 95p - 60 &= -130p + 660 \\ 95p + 130p &= 660 + 60 \\ 225p &= 720 \\ p &= \frac{720}{225} \\ p &= 3.2 \end{aligned}$$

It is fine to write $\frac{720}{225}$ as a decimal here because the decimal terminates (it is exactly 3.2), so nothing is lost to rounding.

Substitute $p = 3.2$ into Ⓐ to find n.

$$\begin{aligned} n &= 95p - 60 \\ n &= 95(3.2) - 60 \\ n &= 304 - 60 \\ n &= 244 \end{aligned}$$

Check in Ⓑ, the equation that was not just used:
$-130(3.2) + 660 = -416 + 660 = 244$.
Both equations give 244, so the solution is correct.

Therefore, at a price of \$3.20 for each doll, supply and demand will be equal, at 244 dolls per week at each store.

This matches the estimate from the graph in question 3e. The graph gives an estimate; the algebra gives the exact answer. The equations can also be used to check the other graph readings: at \$2.50 the gap is $335 - 177.5 = 157.5$ dolls, and at \$3.80 it is $301 - 166 = 135$ dolls.