Warmup: Thumb Length and Palm Area

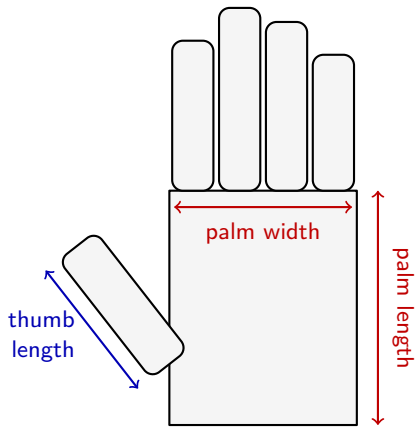
Adapted from Principles of Mathematics 10, McGraw-Hill Ryerson, 2007.

Work with a partner. Measure in centimetres, to the nearest 0.1 cm. Your palm is not really a rectangle, so its area will be a rough estimate, and that is fine.

Measure. Use your writing hand each time. Hold it flat, fingers together.

- **Thumb length:** from the lowest crease on the inside of your thumb, where it bends nearest your palm, to the tip.
- **Palm length:** from the crease at your wrist to the crease at the bottom of your middle finger.
- **Palm width:** straight across your palm at the bottom of your fingers, not counting your thumb.
- **Palm area:** treat your palm as a rectangle, so palm area is palm length $\times$ palm width, in cm^2.

Measure your own writing hand, then your partner's. Record both in the table. Then each of you adds your thumb length and palm area to the class table on the board.

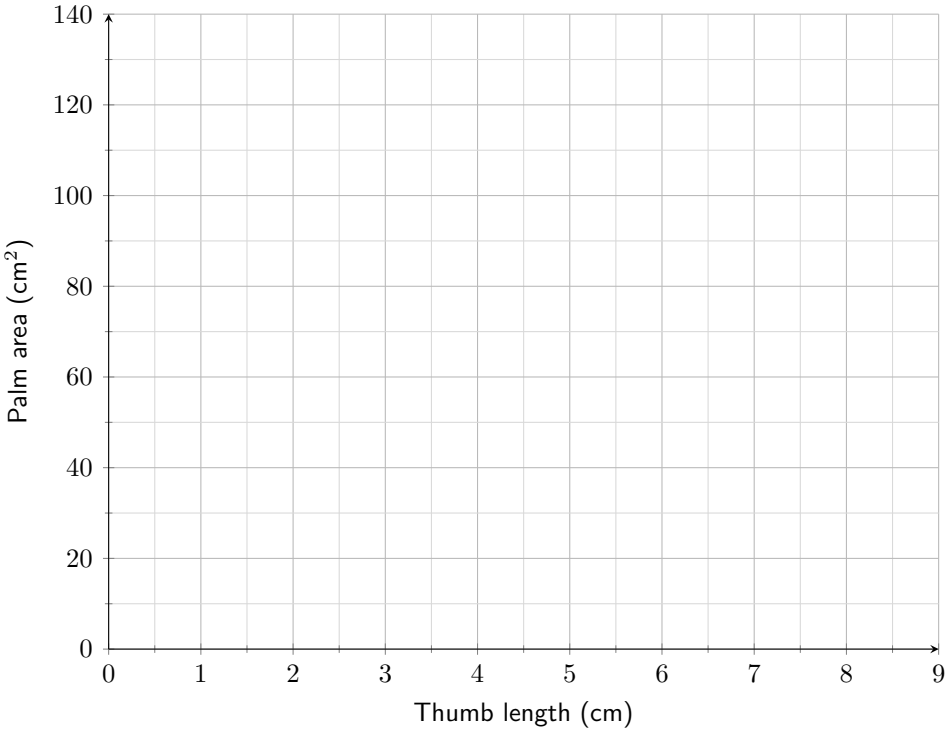


Who	Thumb length (cm)	Palm length (cm)	Palm width (cm)	Palm area (cm^2)
You				
Your partner				
Anyone else at your table				
Anyone else at your table				

On the board: a two-column table, Thumb length (cm) and Palm area (cm^2), one row per person, to one decimal place.

1. **Before you turn the page:** Which quantity would you like to predict from the other? Which quantity goes on the horizontal axis, and why?

2. Plot. Plot the class data from the board on the grid.



3. Describe. Describe the relationship between thumb length and palm area in words.

4. Draw. Draw one smooth curve of best fit, with about as many points above it as below it.

5. Why a curve? Lay a ruler along your points and extend it to the left. Where does it cross the vertical axis? What would that mean? Why might a curve suit these data better than a straight line?

6. Predict. Someone in another class has an 8.1 cm thumb. Use your curve to predict their palm area.

Show your reading on the graph. Predicted palm area: _____ cm^2

7. Look back. Neither quantity here was controlled by anyone. What decided the axes?

Warmup: Thumb Length and Palm Area — Sample Answers

Adapted from Principles of Mathematics 10, McGraw-Hill Ryerson, 2007.

The real answers come from the measurements made in the room. The class table and graph below are *sample data* to show what a complete warmup looks like. Answers in words are model sentences; other answers that are well argued are fine.

Sample data. The table as one table group might fill it in, and below it the class table from the board, whose points are plotted in question 2. Each palm area is palm length × palm width, rounded to the nearest $0.1\,\text{cm}^2$.

Who	Thumb length (cm)	Palm length (cm)	Palm width (cm)	Palm area (cm^2)
You	5.3	8.8	6.3	55.4
Your partner	5.5	9.0	6.5	58.5
Anyone else at your table	6.0	9.8	7.2	70.6
Anyone else at your table	6.2	10.0	7.2	72.0

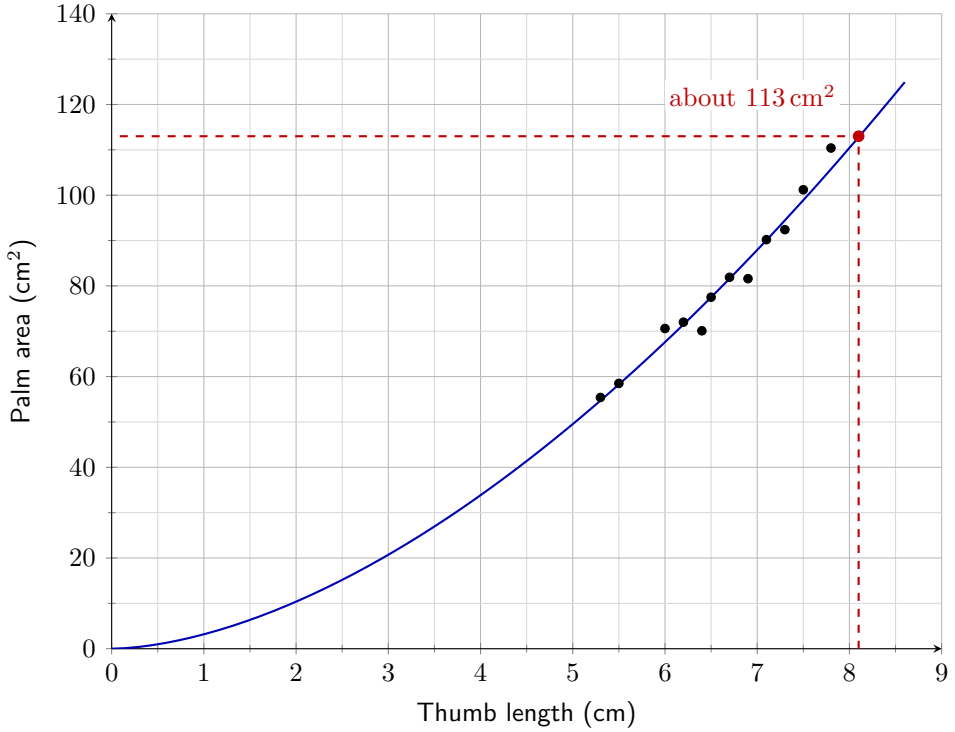
Thumb length (cm)	5.3	5.5	6.0	6.2	6.4	6.5	6.7	6.9	7.1	7.3	7.5	7.8
Palm area (cm^2)	55.4	58.5	70.6	72.0	70.1	77.5	81.9	81.6	90.2	92.4	101.2	110.4

1. **Before you turn the page:** Which quantity would you like to predict from the other? Which quantity goes on the horizontal axis, and why?

   I want to estimate palm area from something quick to measure, so I predict palm area from thumb length (a thumb is one measurement; a palm area is two and a calculation). The quantity I predict *from* (thumb length) goes on the horizontal axis; the quantity I predict (palm area) goes on the vertical axis.

   The printed grid on page 2 shows this convention, which the Discussion will justify. A student who argued for the other way round, with a reason, has still answered the question.

2. Plot. Sample data, a curve of best fit (question 4), and the 8.1 cm reading (question 6).



3. Describe. Describe the relationship between thumb length and palm area in words.

As thumb length increases, palm area increases. The points lie close to a clear trend, so the relationship is fairly strong. The trend bends upward: an extra centimetre of thumb adds more palm area to a long thumb than to a short one.

4. Draw. A smooth curve through the middle of the points, as on the graph in question 2. Extended to the left, it should head toward $(0, 0)$, since no thumb means no hand.

5. Why a curve? Lay a ruler along your points and extend it to the left. Where does it cross the vertical axis? What would that mean? Why might a curve suit these data better than a straight line?

Extended to the left, a ruler along the sample points crosses the vertical axis near $-60\,\text{cm}^2$: a 0 cm thumb with a negative palm area, which is impossible. The points bend upward instead. As a hand gets bigger, its palm gets both longer *and* wider, so the area (length × width) grows faster than the thumb length does.

6. Predict. Someone in another class has an 8.1 cm thumb. Use your curve to predict their palm area. Show your reading on the graph.

About $113\,\text{cm}^2$ (reading on the graph). Accept 108 to $125\,\text{cm}^2$: the range spans a straight-line reading (about 111) to a steeper curve (about 120). The longest thumb in the data is 7.8 cm, so this reaches beyond the data and is less certain.

7. Look back. Neither quantity here was controlled by anyone. What decided the axes?

Nobody set anyone's thumb length. When nothing is controlled, the prediction you want to make decides: the quantity you predict *from* is the independent variable (horizontal axis), and the quantity you predict is the dependent variable (vertical axis).